Adjoint-accelerated Bayesian Inference in a 3D Helmholtz Solver for Thermoacoustic Test Rigs

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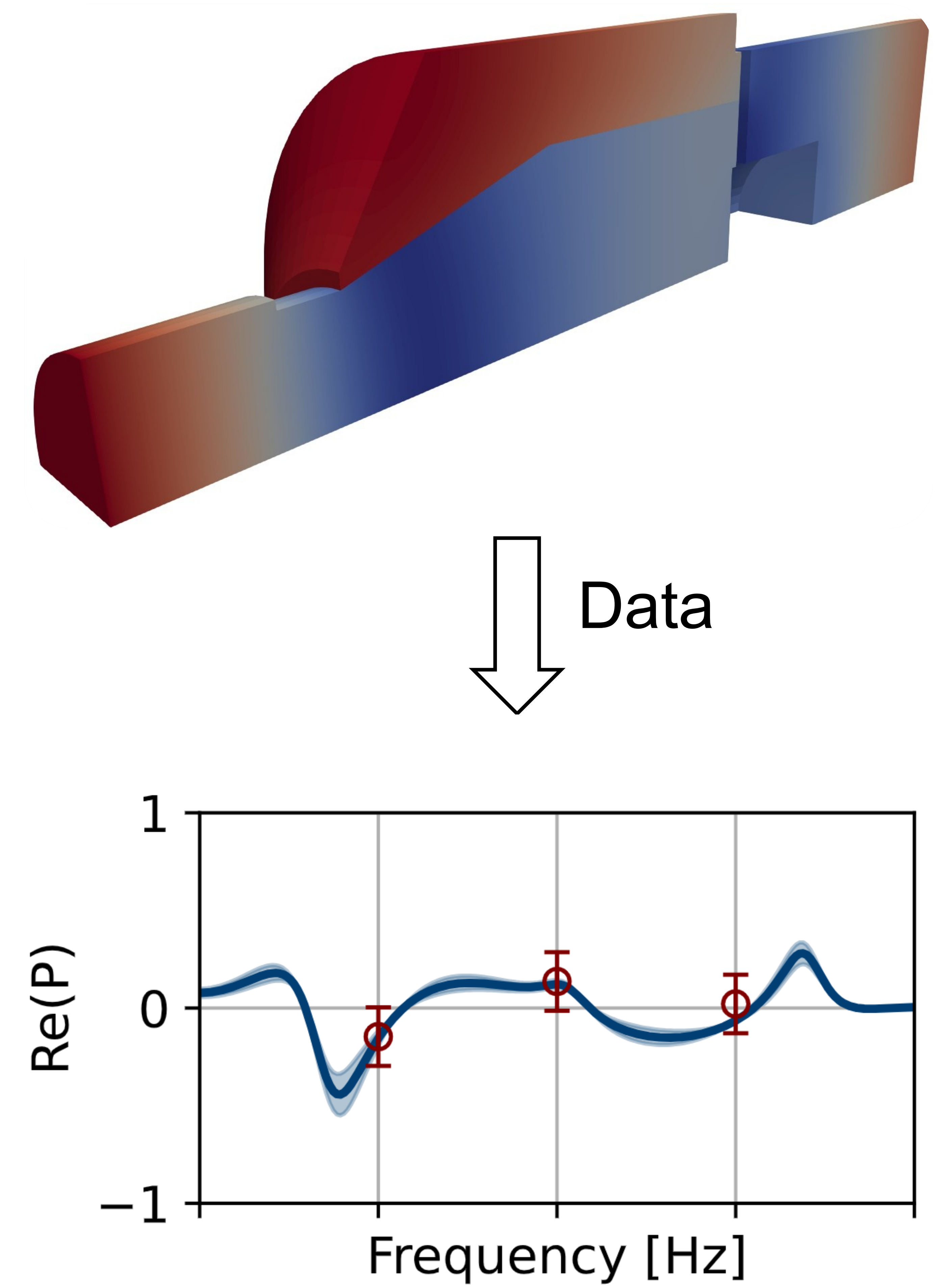
Brian Pitt Excellence Award, 08/07/2025

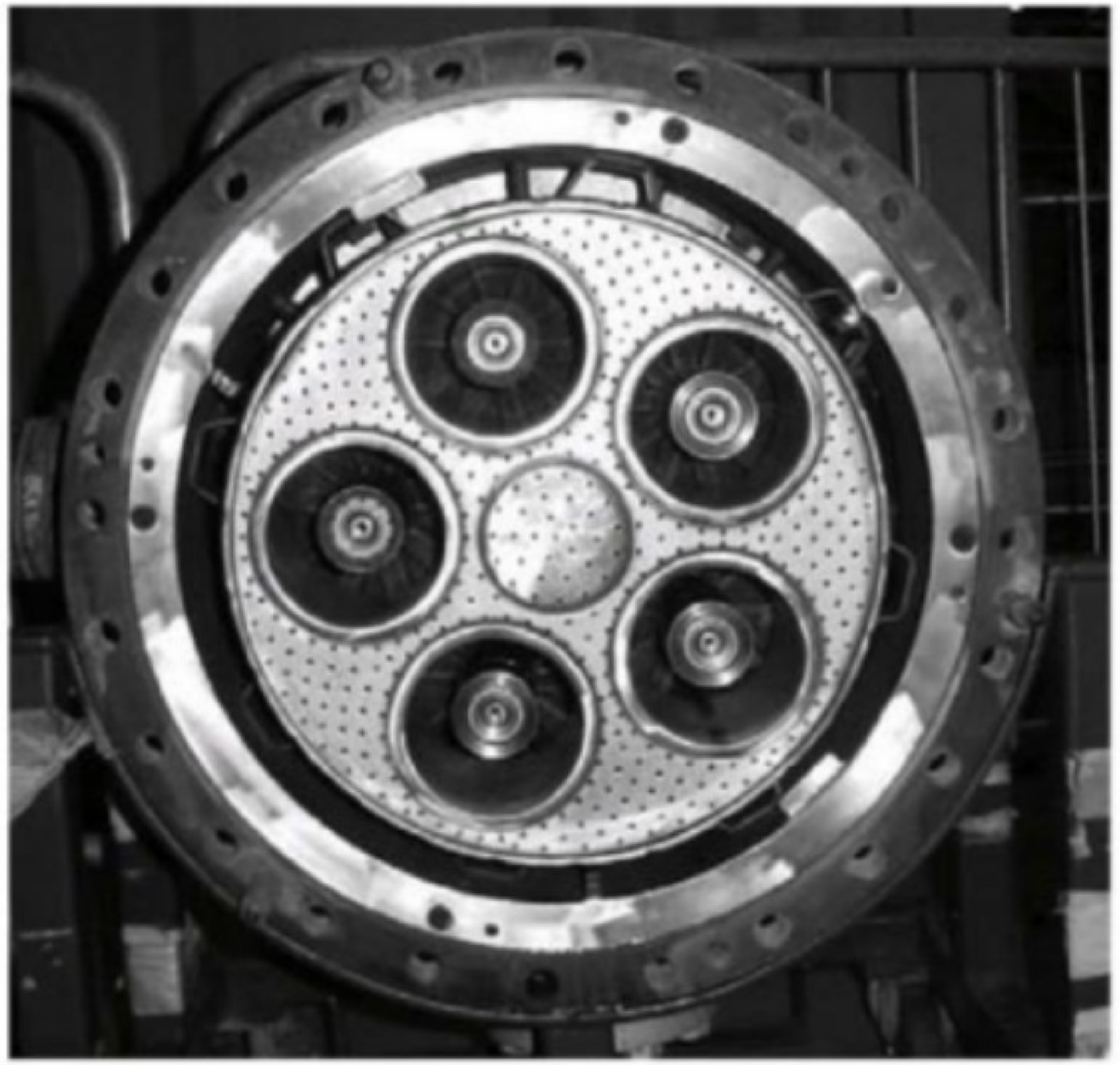


<u>Collaborative Research Project:</u>

- TUM – PD Dr. habil Camilo Silva
- University of Cambridge
  Prof. M. Juniper & M. Yoko, Ph. D.
- Rolls-Royce Deutschland

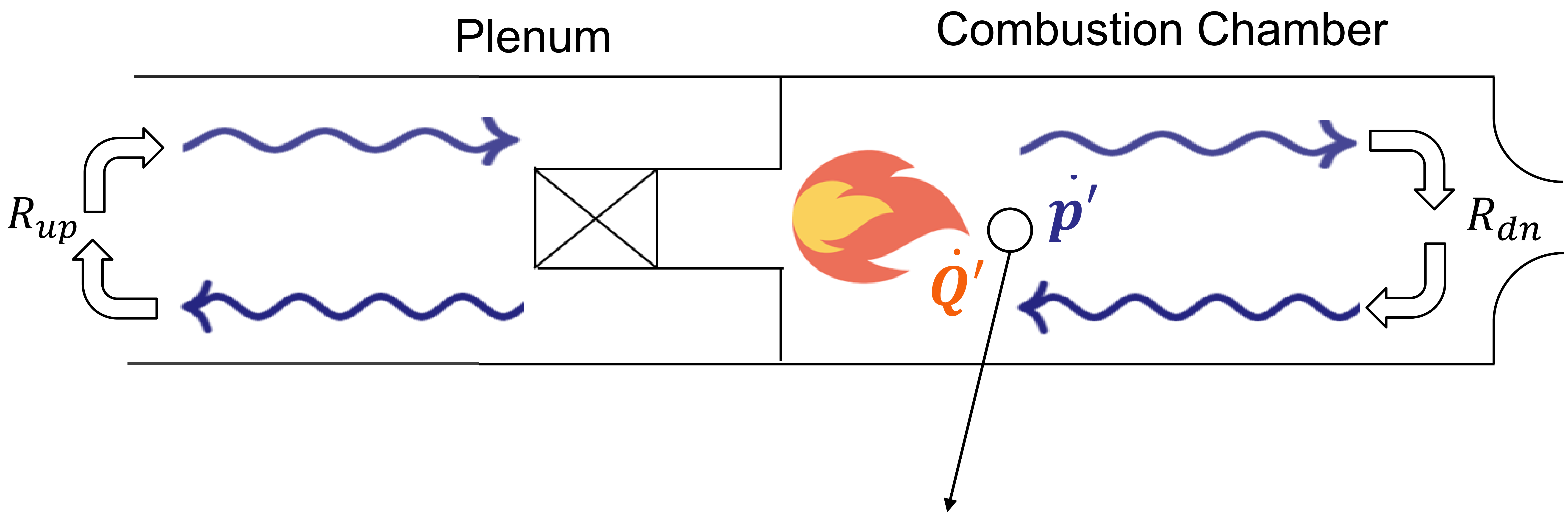
Thermoacoustic instabilities have to be mitigated

Thermoacoustic Instability

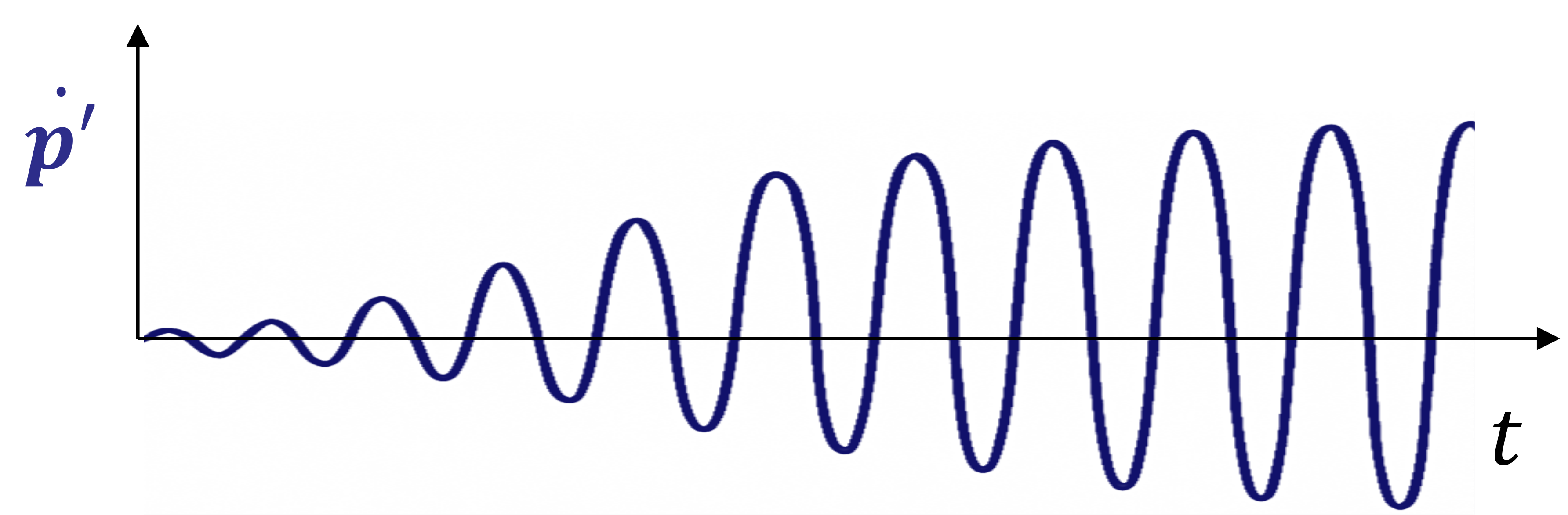


Goy et al. 2006

The flame excites acoustic pressure modes in a feedback loop



if $\int \dot{p}'\dot{Q}'dt > 0$ ➔ Excitation



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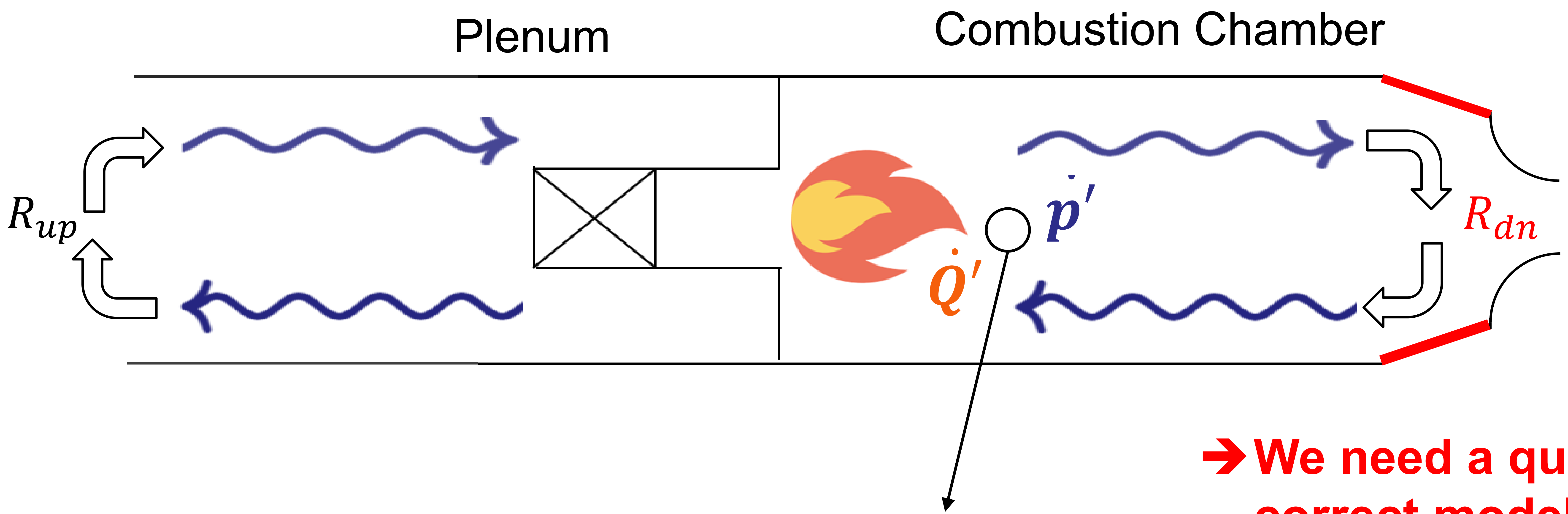
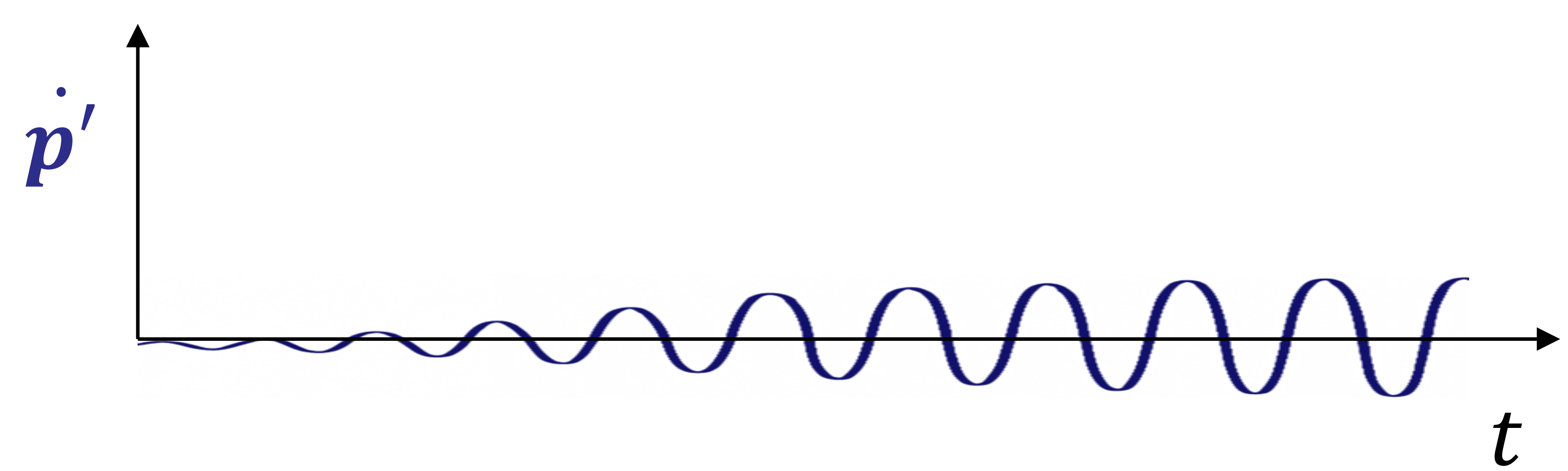


➔ **We need a quantitatively correct model**

if $\int \dot{p}'\dot{Q}'dt > 0$ ➔ Excitation

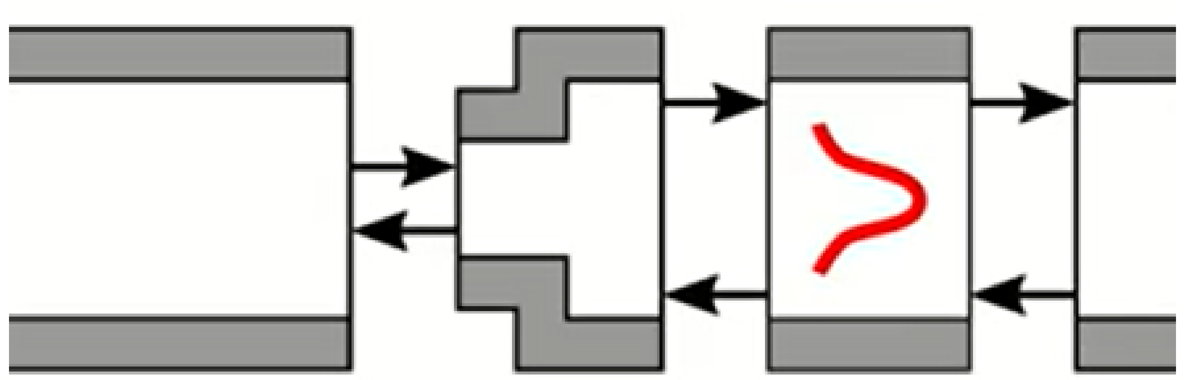


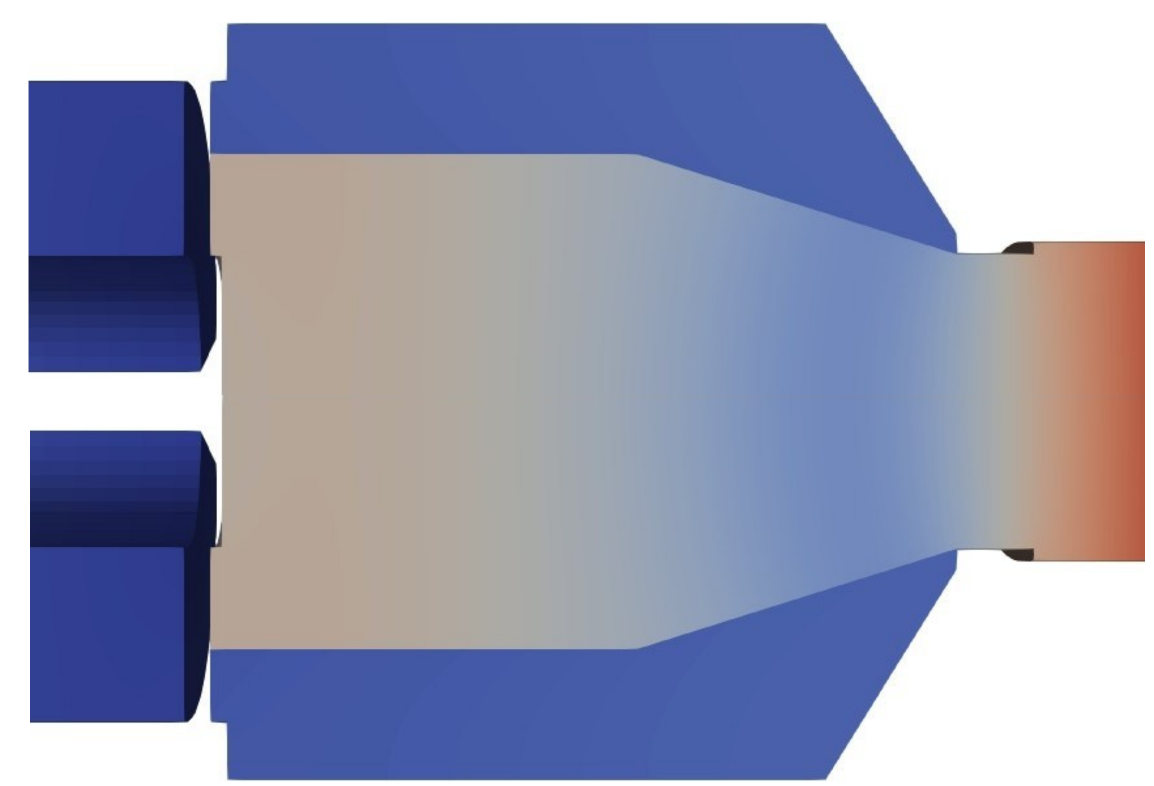
Qualitatively correct models cannot predict thermoacoustic instabilities and their mitigation strategies.

Complexity & Cost

$	$$	$$$$
Low Order Modeling (LOM)	**Helmholtz-Solver**	Large Eddy Simulation (LES)

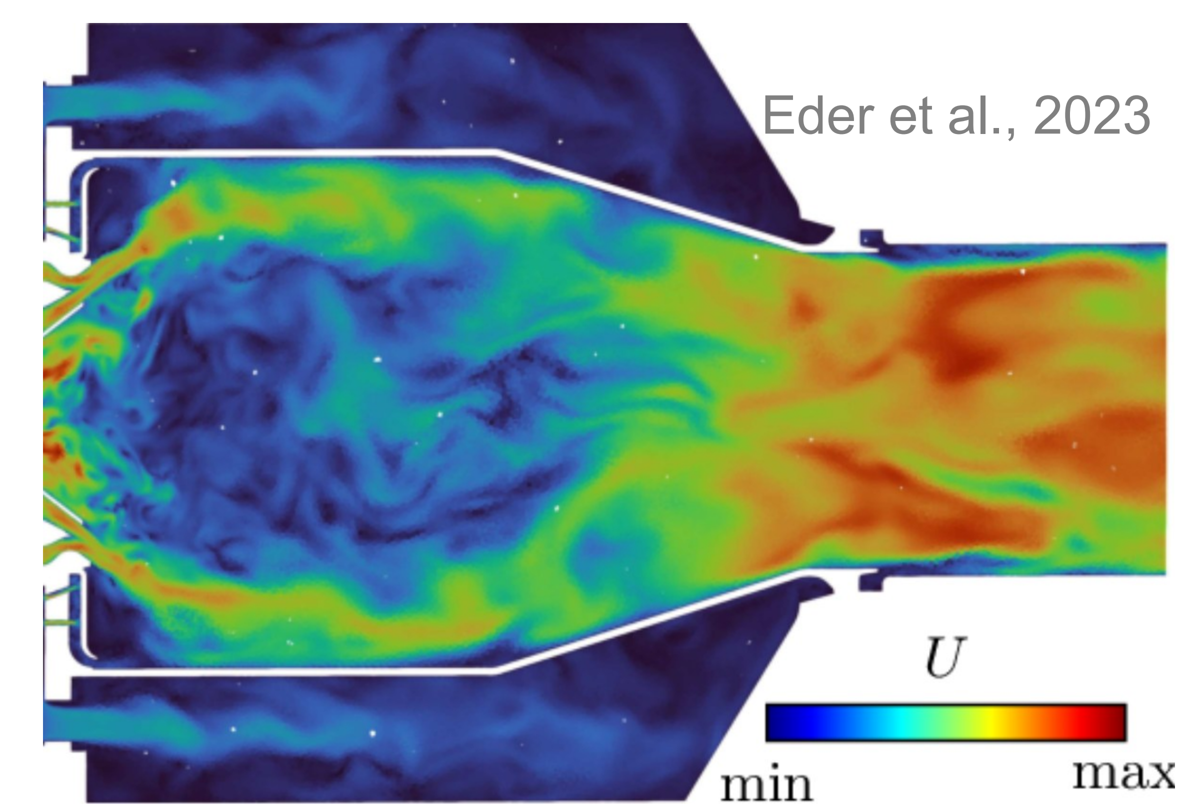
Emmert et al., 2017





$$\omega^2 \hat{p} + \nabla \cdot (\bar{c}^2 \nabla \hat{p}) = i\omega(\gamma - 1)\hat{q}$$

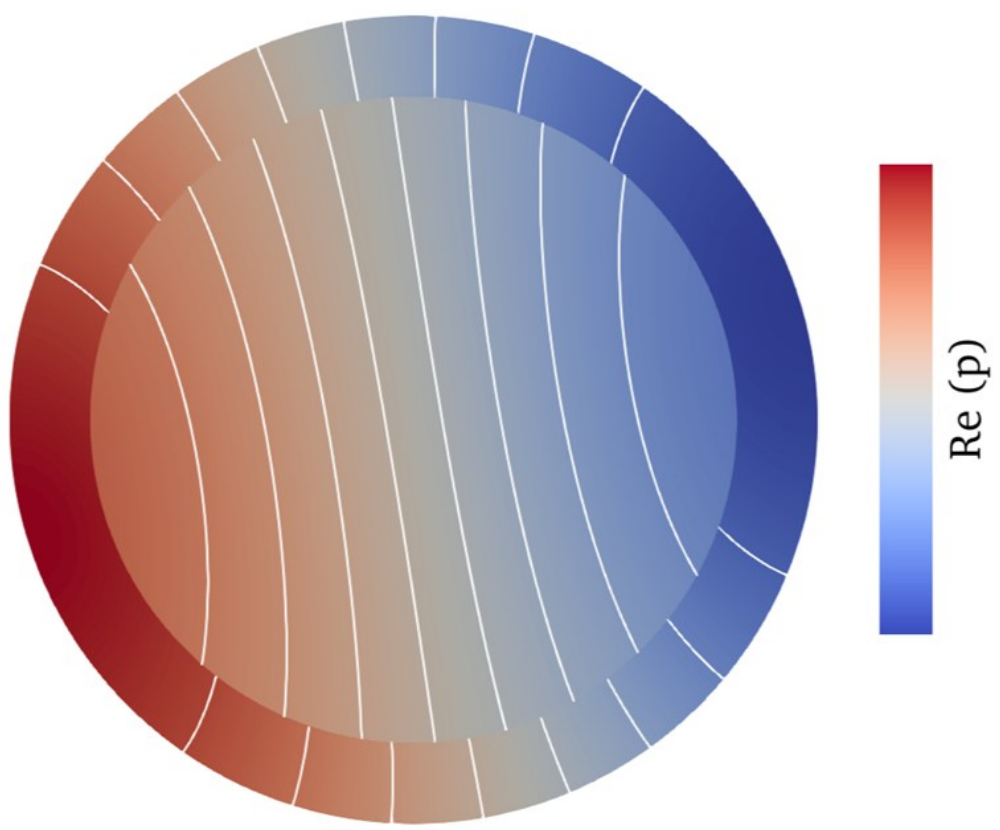
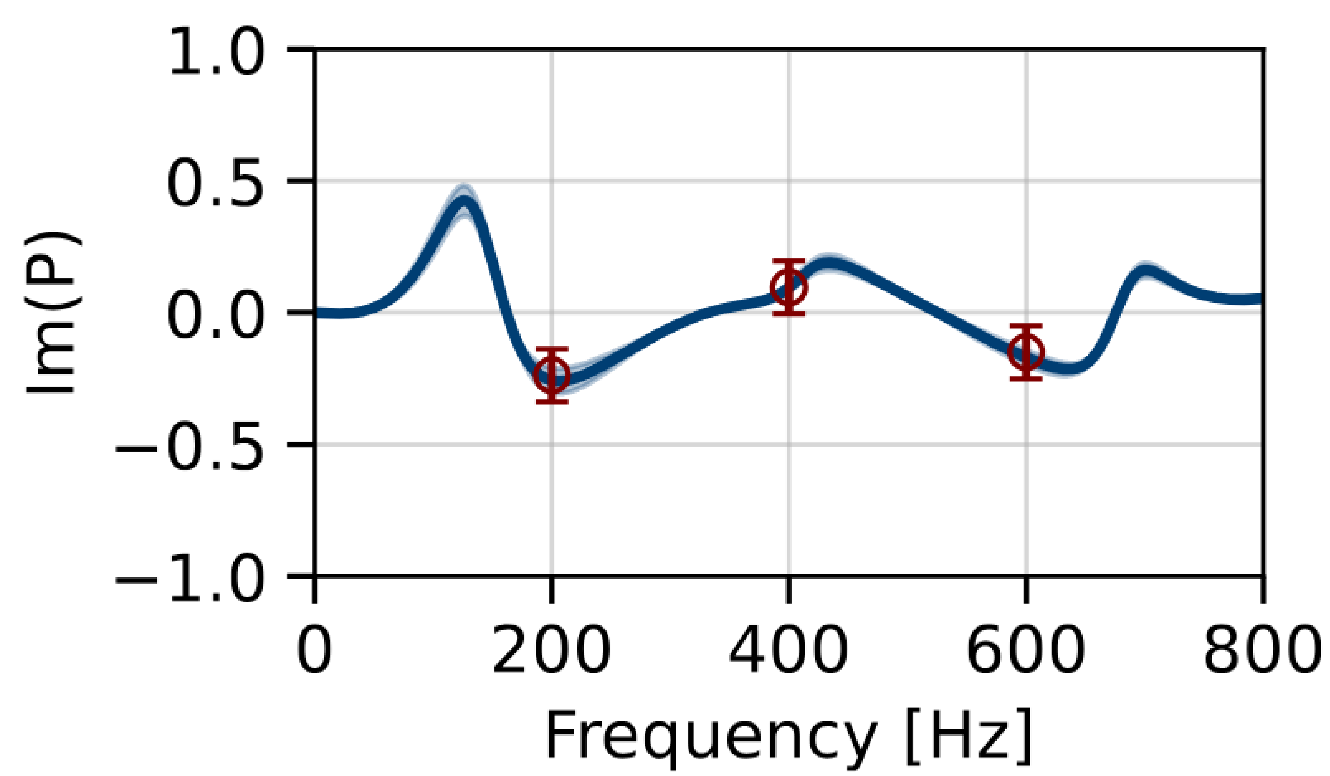
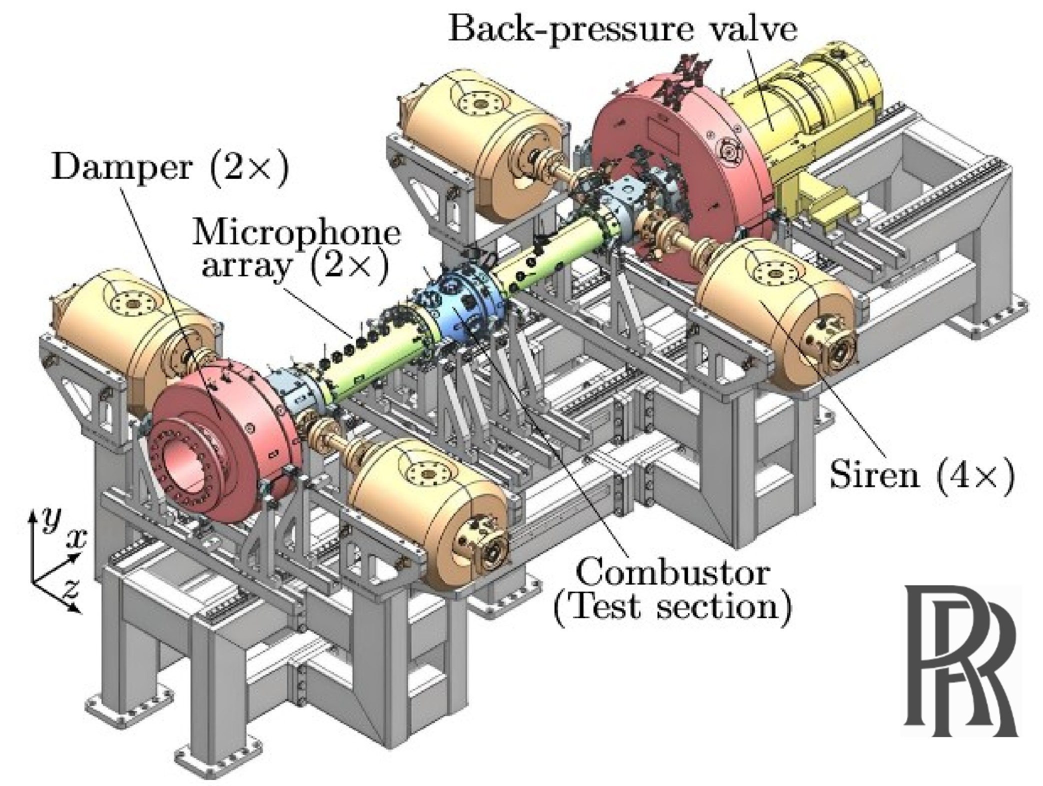
$$-i\omega \hat{\boldsymbol{u}} + \frac{1}{\bar{\rho}} \nabla \hat{p} = \frac{1}{\bar{\rho}} \hat{\boldsymbol{f}}\,.$$



Can we create a **quantitatively** correct Helmholtz-Solver if we incorporate **physics-based models** and learn from **experimental data**?

Qualitatively accurate model → ***Quantitatively*** accurate model

Methods | Results

Acoustic Damping: Physcis-based models	Bayesian Inference: Learning from Data	Application: Rolls-Royce SCARLET rig
		

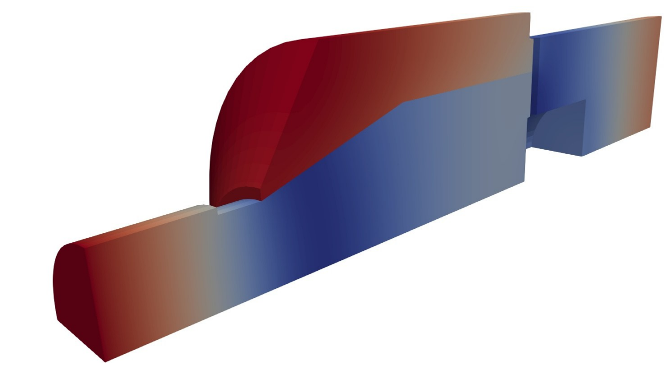
➔ *Novelty in the literature*

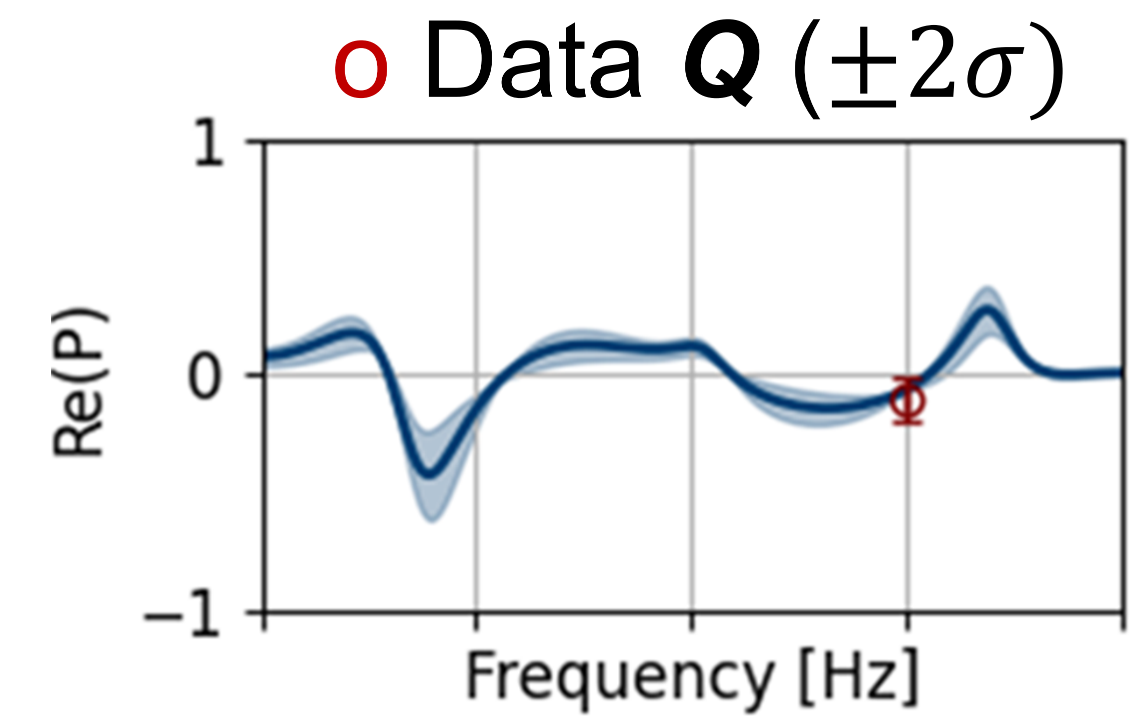
Uncertain parameters are inferred from experimental data using adjoint-accelerated Bayesian Inference

Bayesian posterior of parameters **a**

$$\underbrace{\boxed{P(\boldsymbol{a} \mid \boldsymbol{Q}, \mathscr{H}_i)}}_{\text{posterior}} = \frac{\overbrace{\boxed{P(\boldsymbol{Q} \mid \boldsymbol{a}, \mathscr{H}_i)}}^{\text{likelihood}}\,\overbrace{\boxed{P(\boldsymbol{a} \mid \mathscr{H}_i)}}^{\text{prior}}}{\underbrace{\boxed{P(\boldsymbol{Q} \mid \mathscr{H}_i)}}_{\text{evidence}}}$$

$\mathscr{H}_i$





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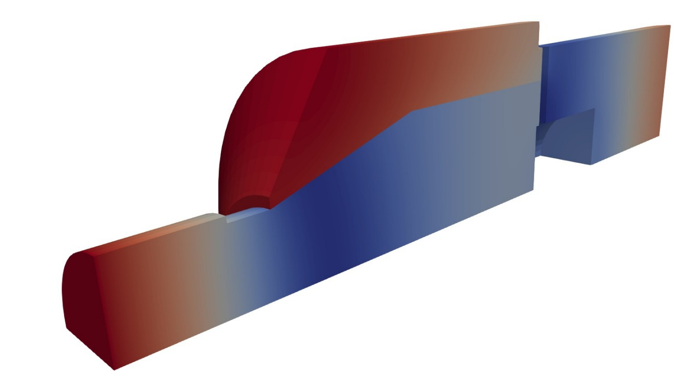
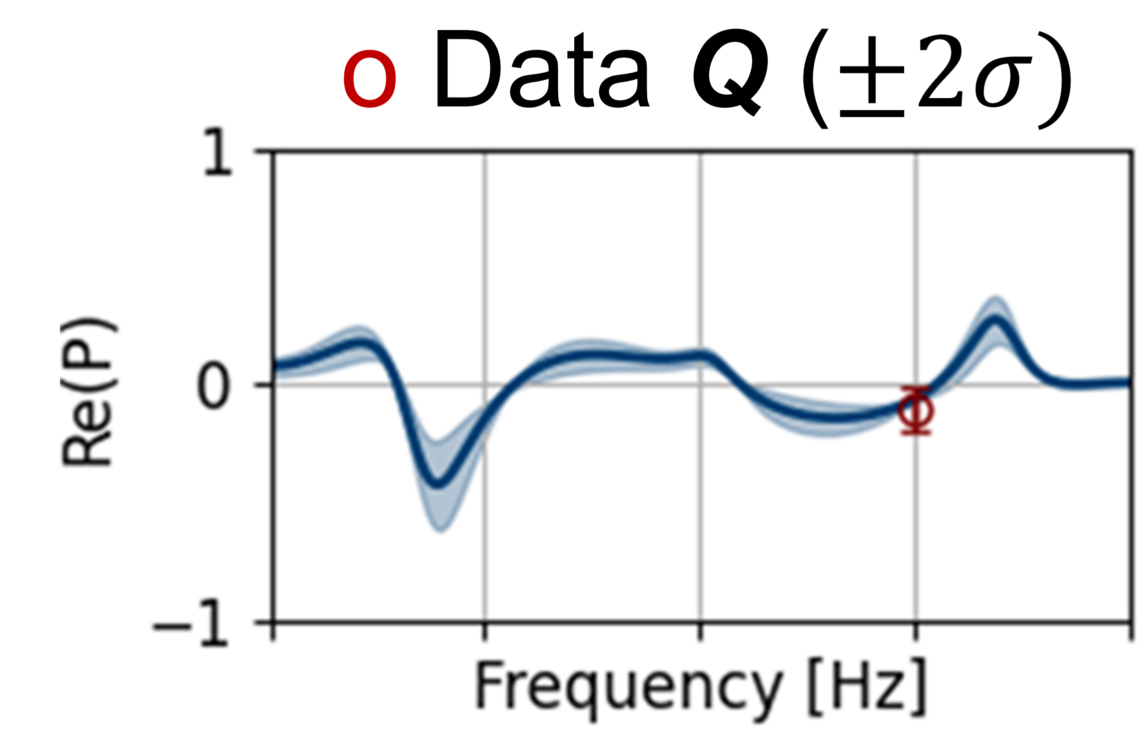
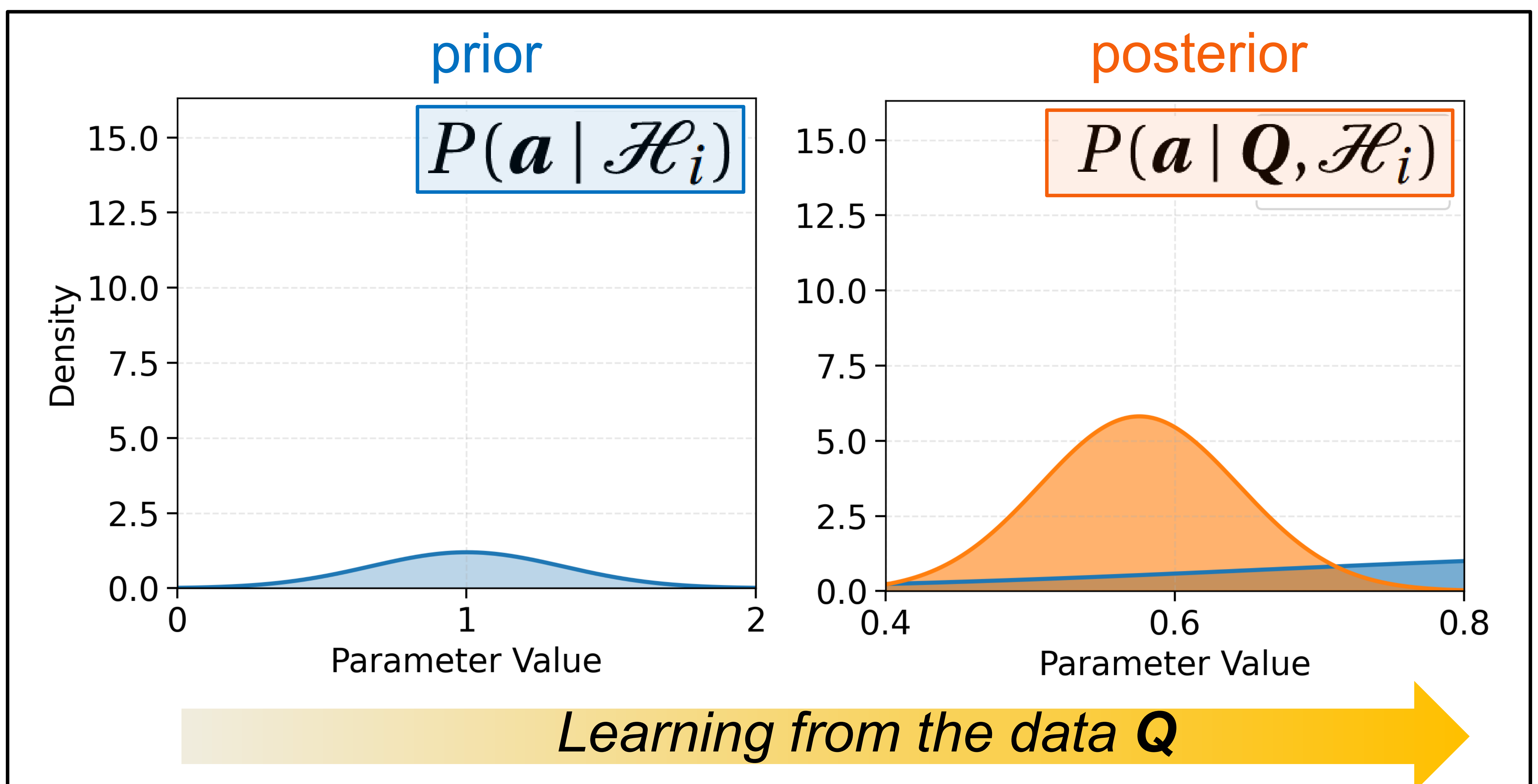
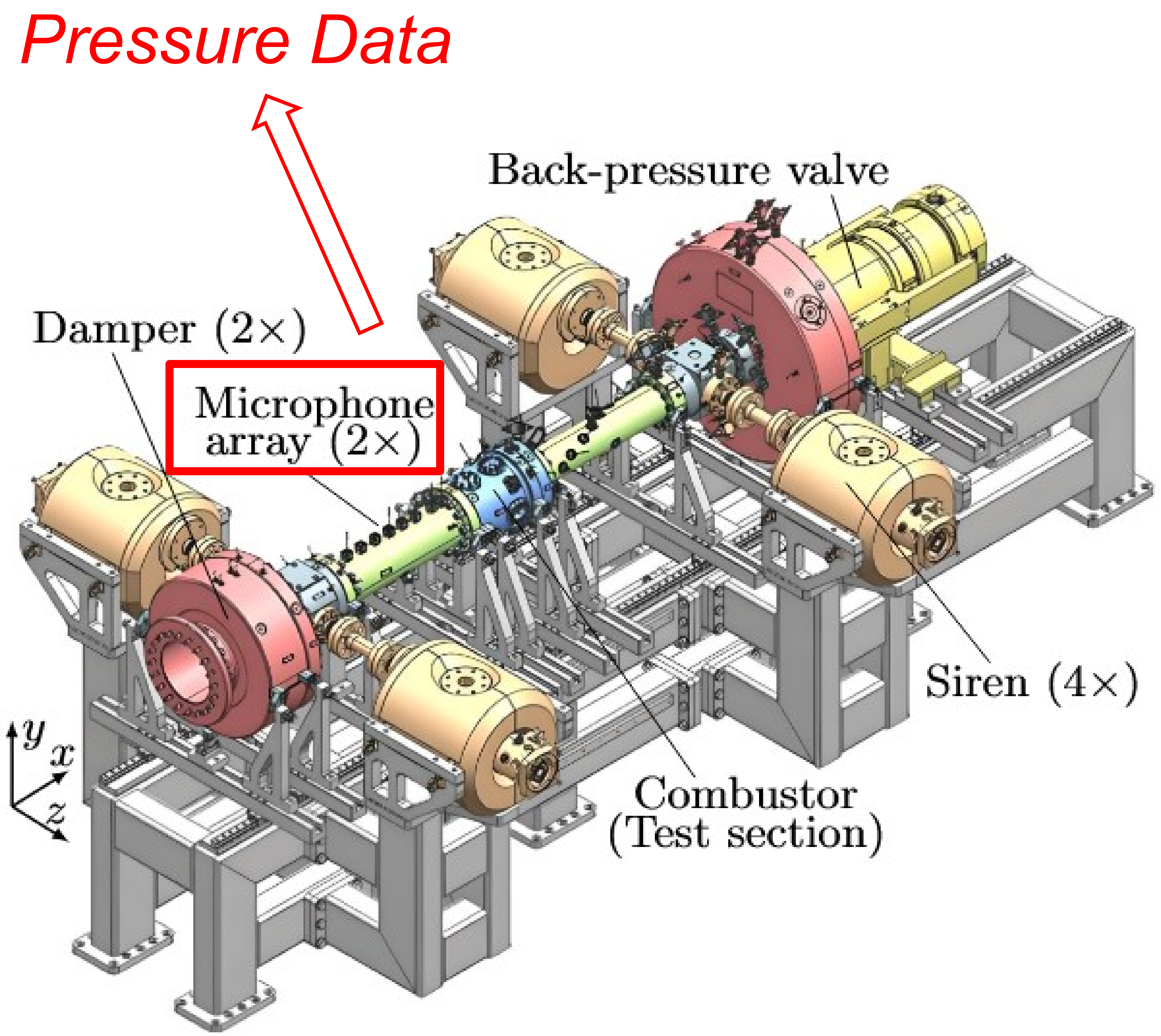






The combustion chamber in RR-SCARLET (25 bar, 950K, 4kg/s) comprises complex flow features



Eder et al., 2024

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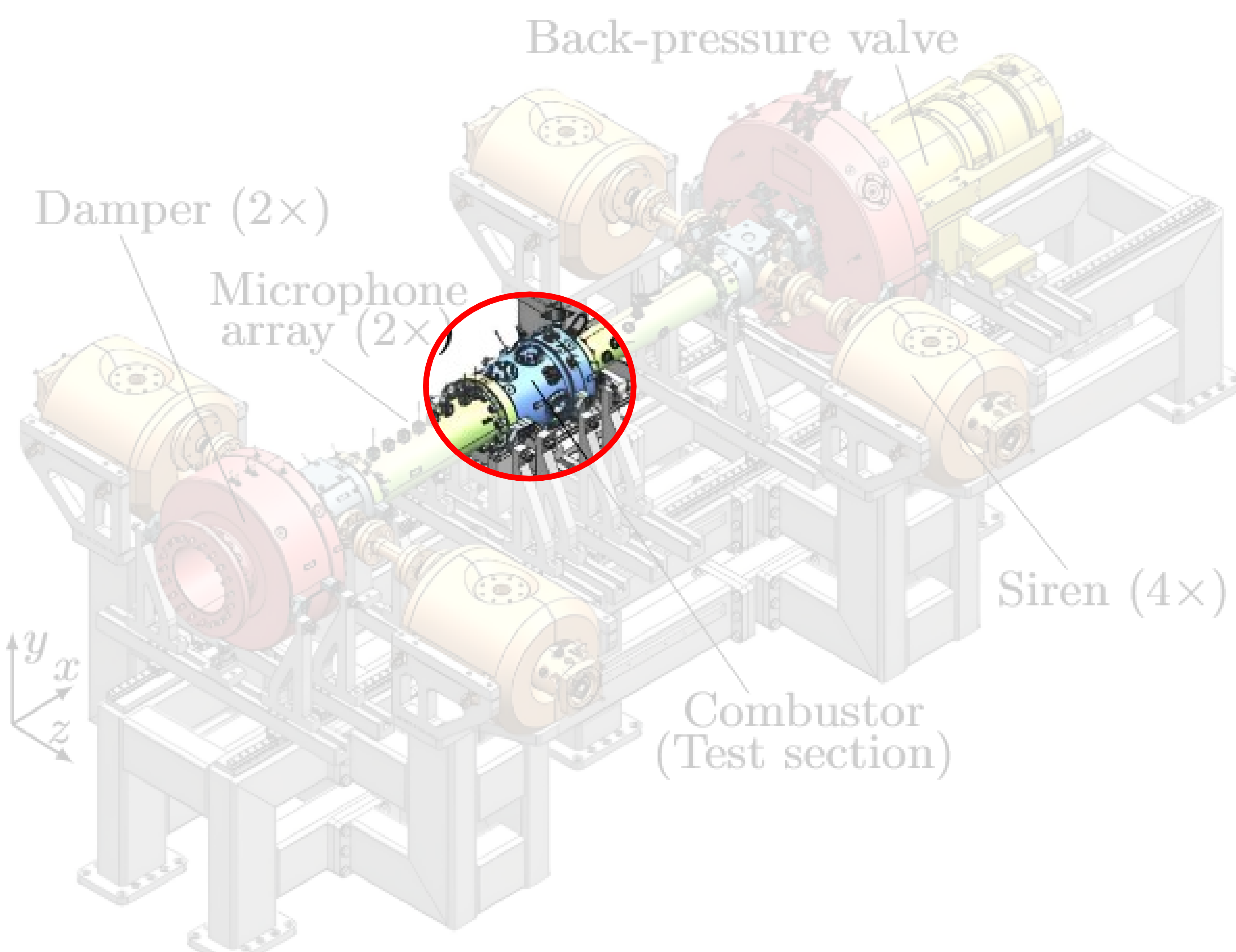

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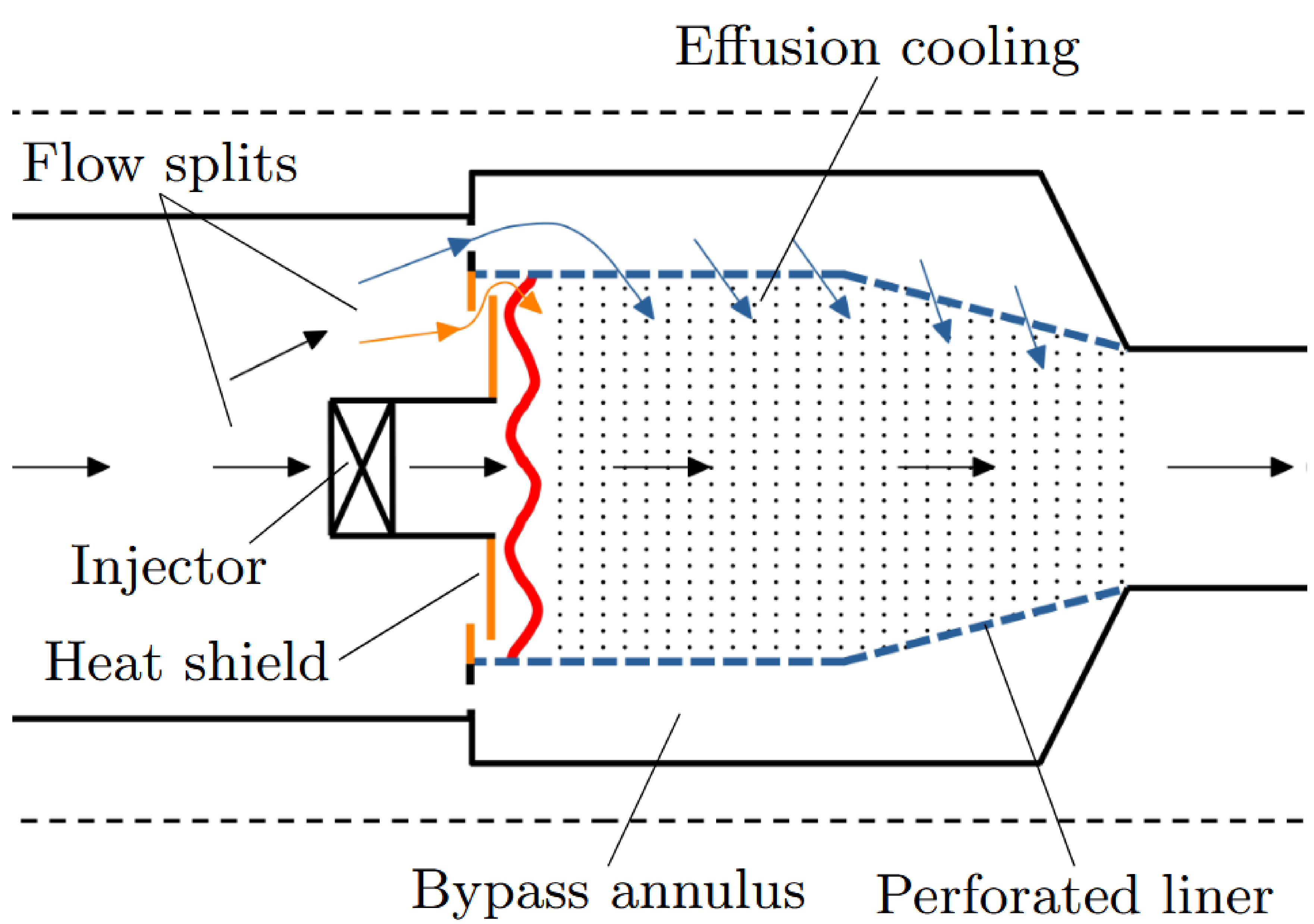

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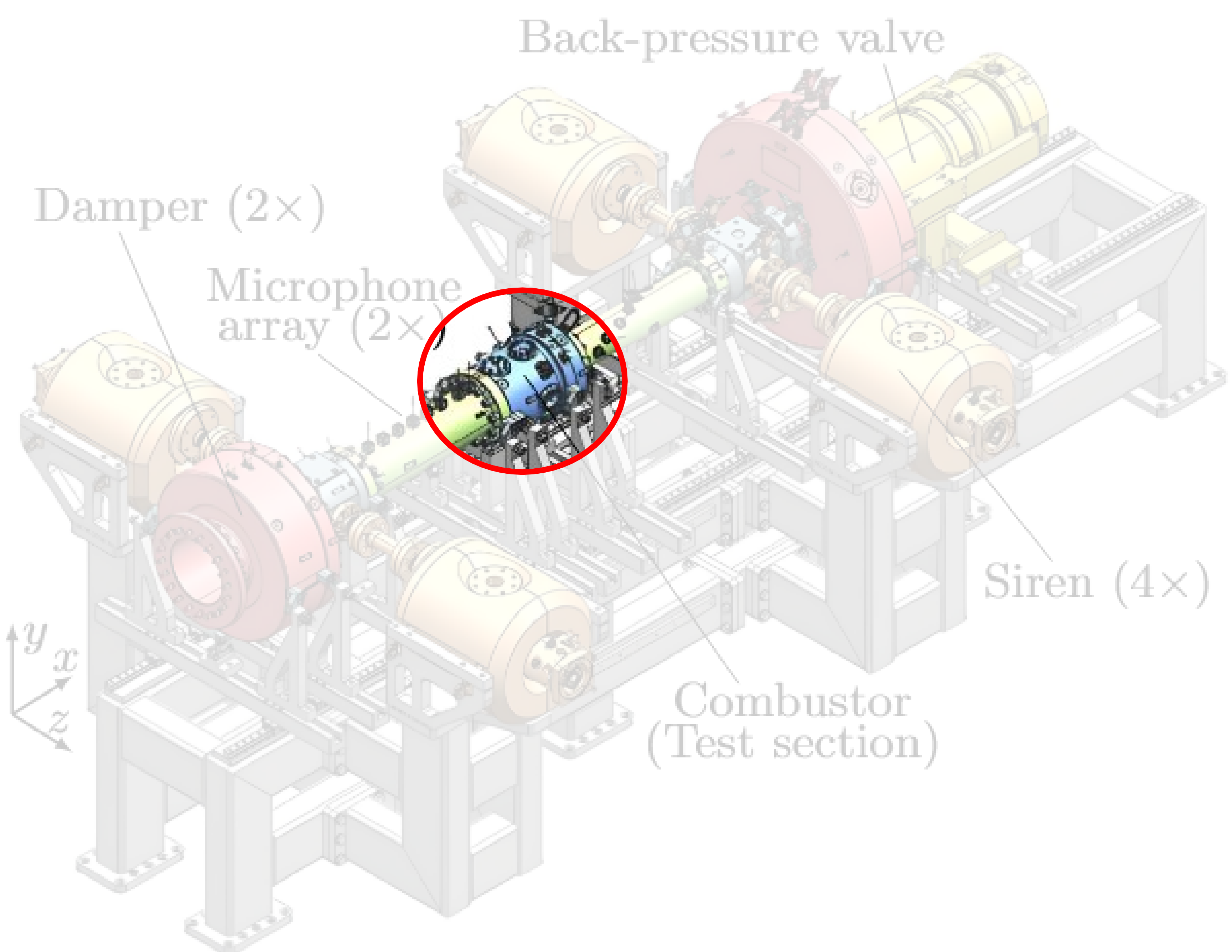
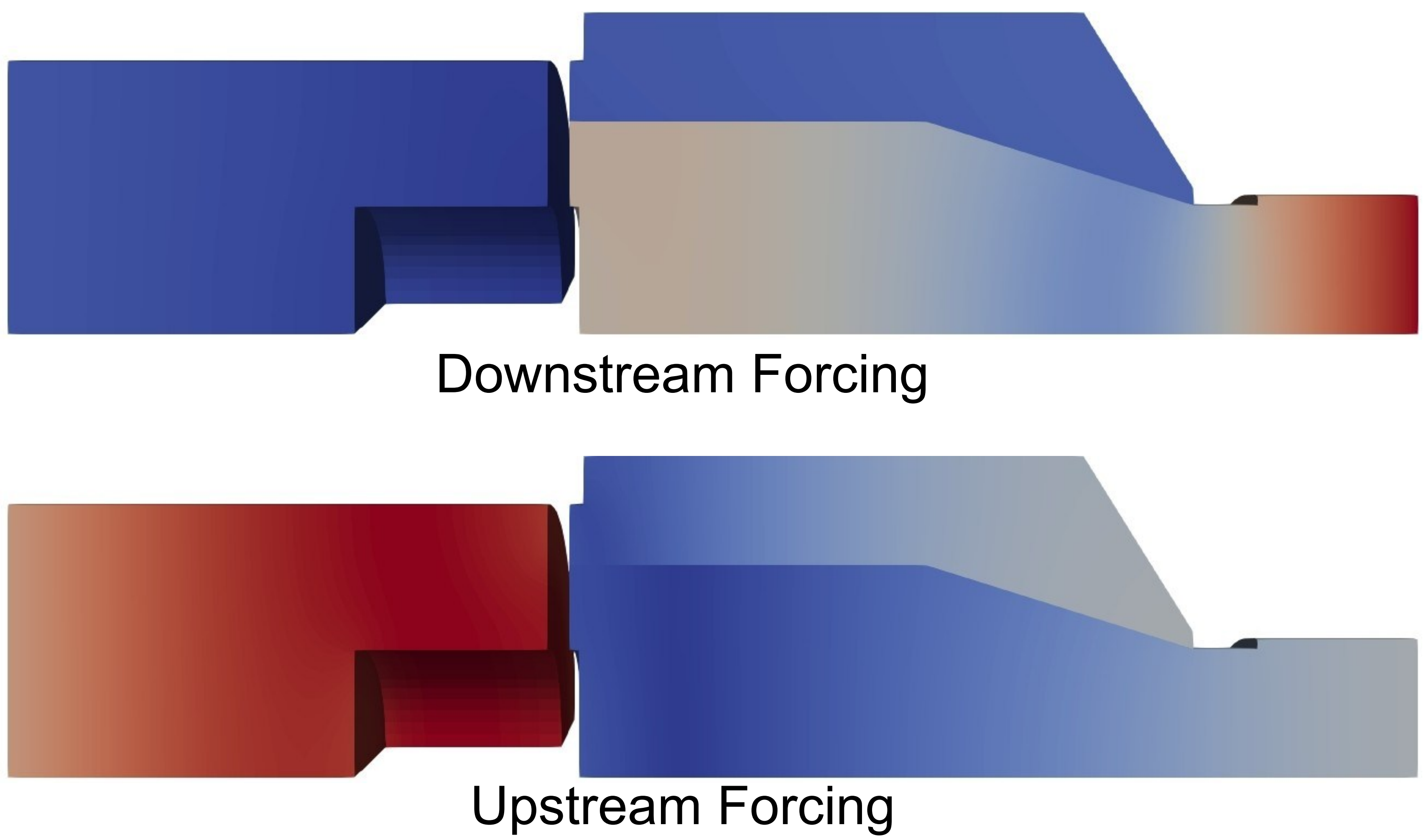
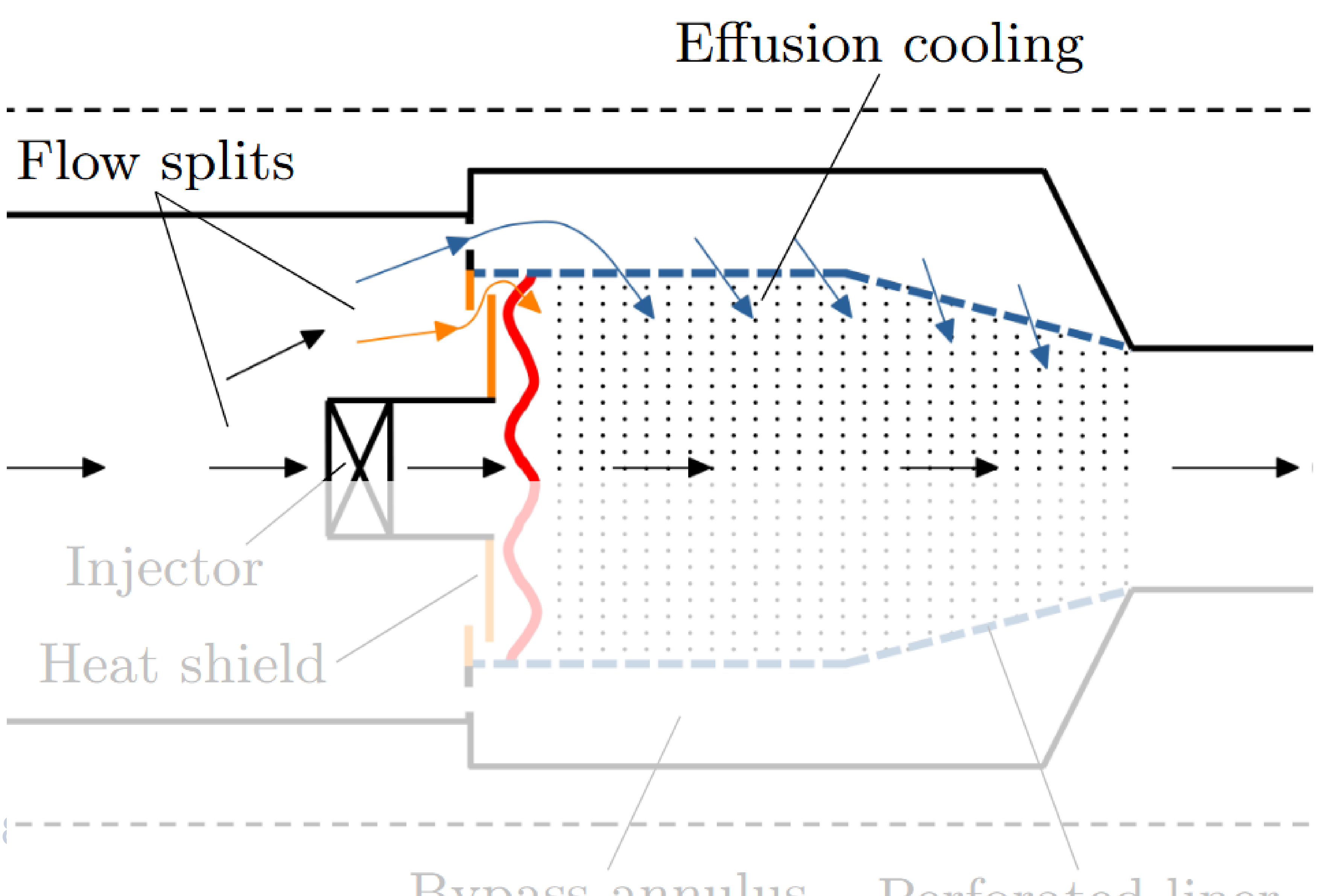


Eder et al., 2024

Helmholtz-solver (+ physics-based models)





Data is assimilated to RR-SCARLET (20 Frequencies, 10 Microphones)

Upstream Mics.

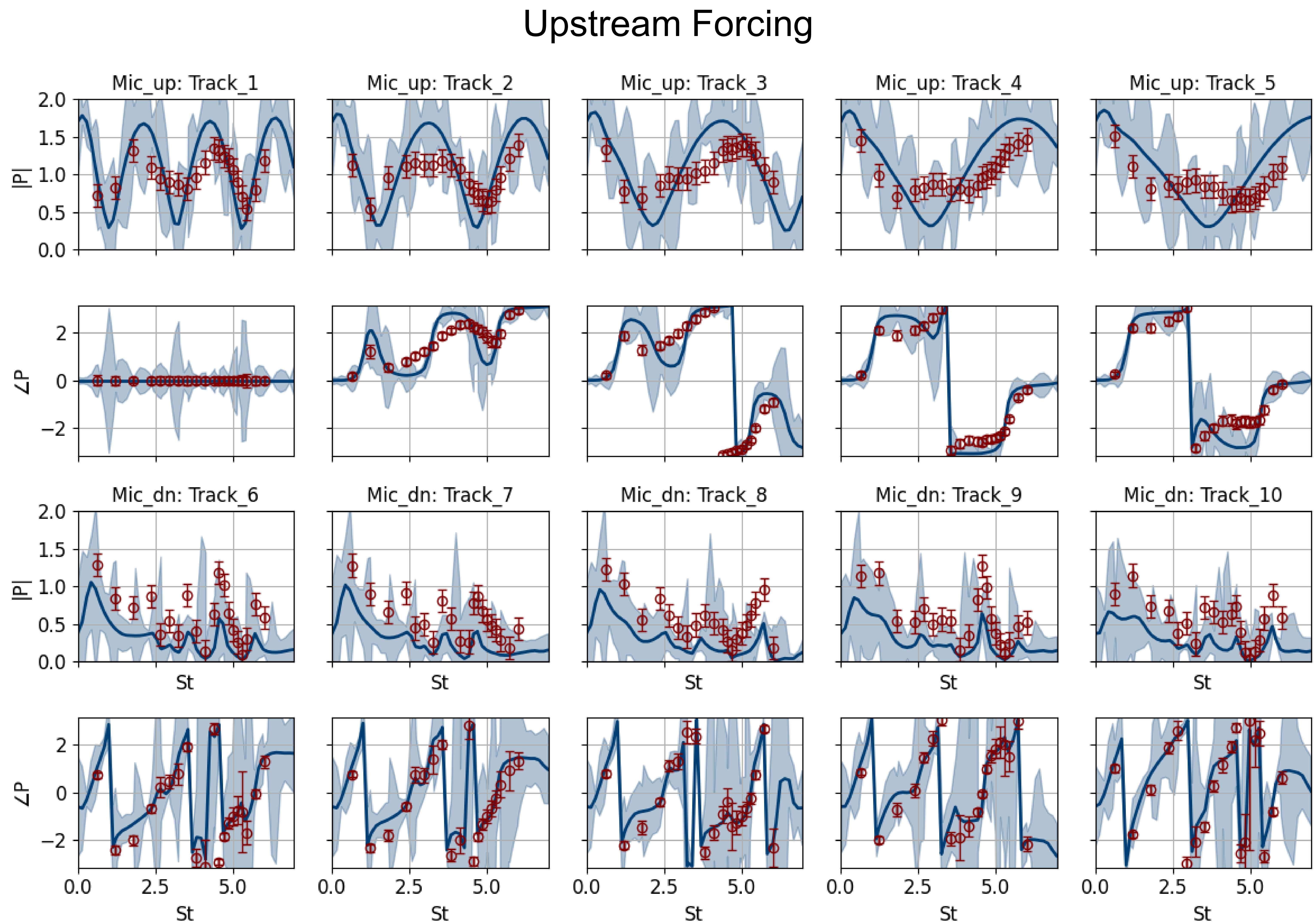
(1 -5)

Downstream Mics.

(6-10)

– Model ($\pm 2\sigma$)

o Data ($\pm 2\sigma$)



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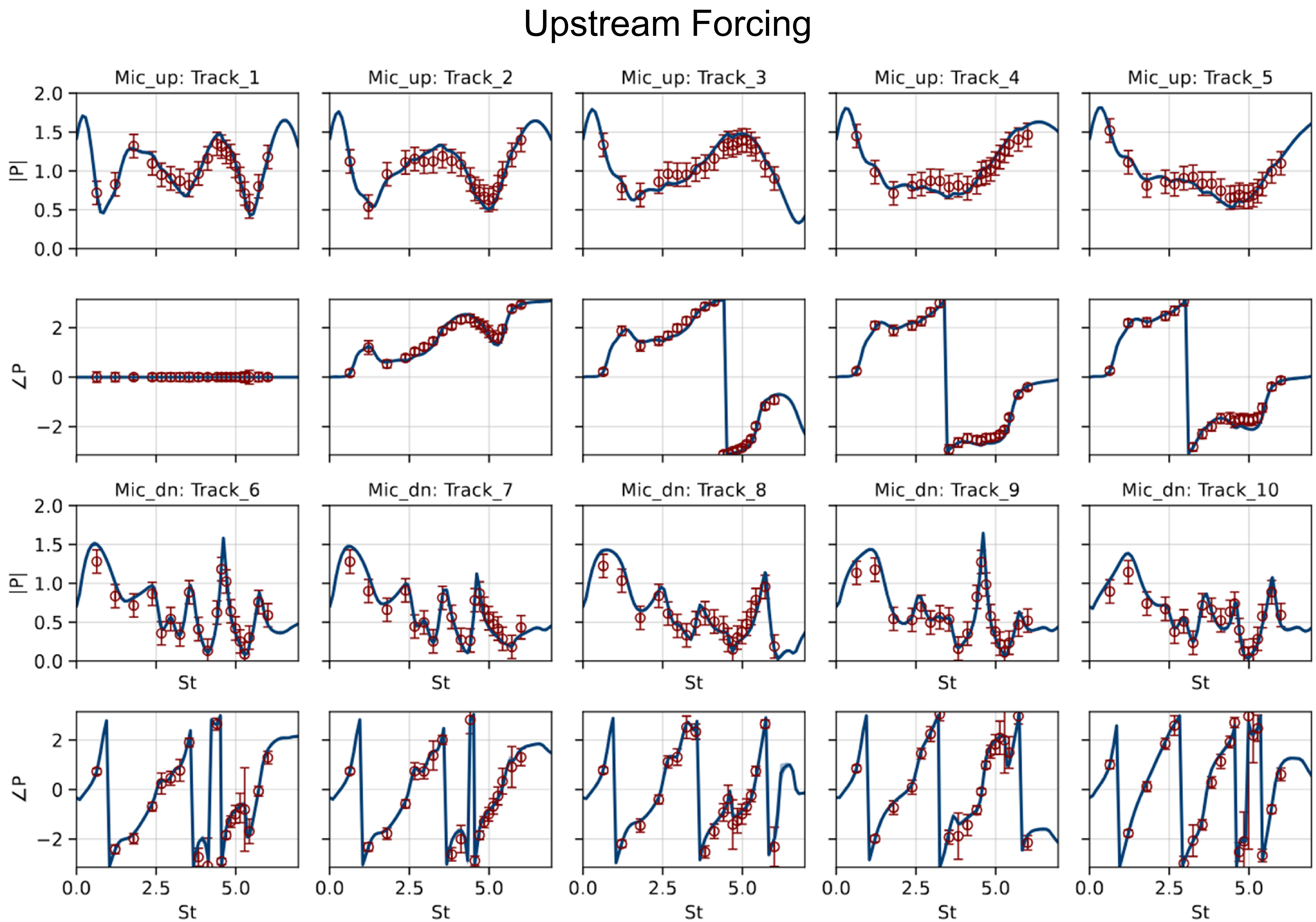

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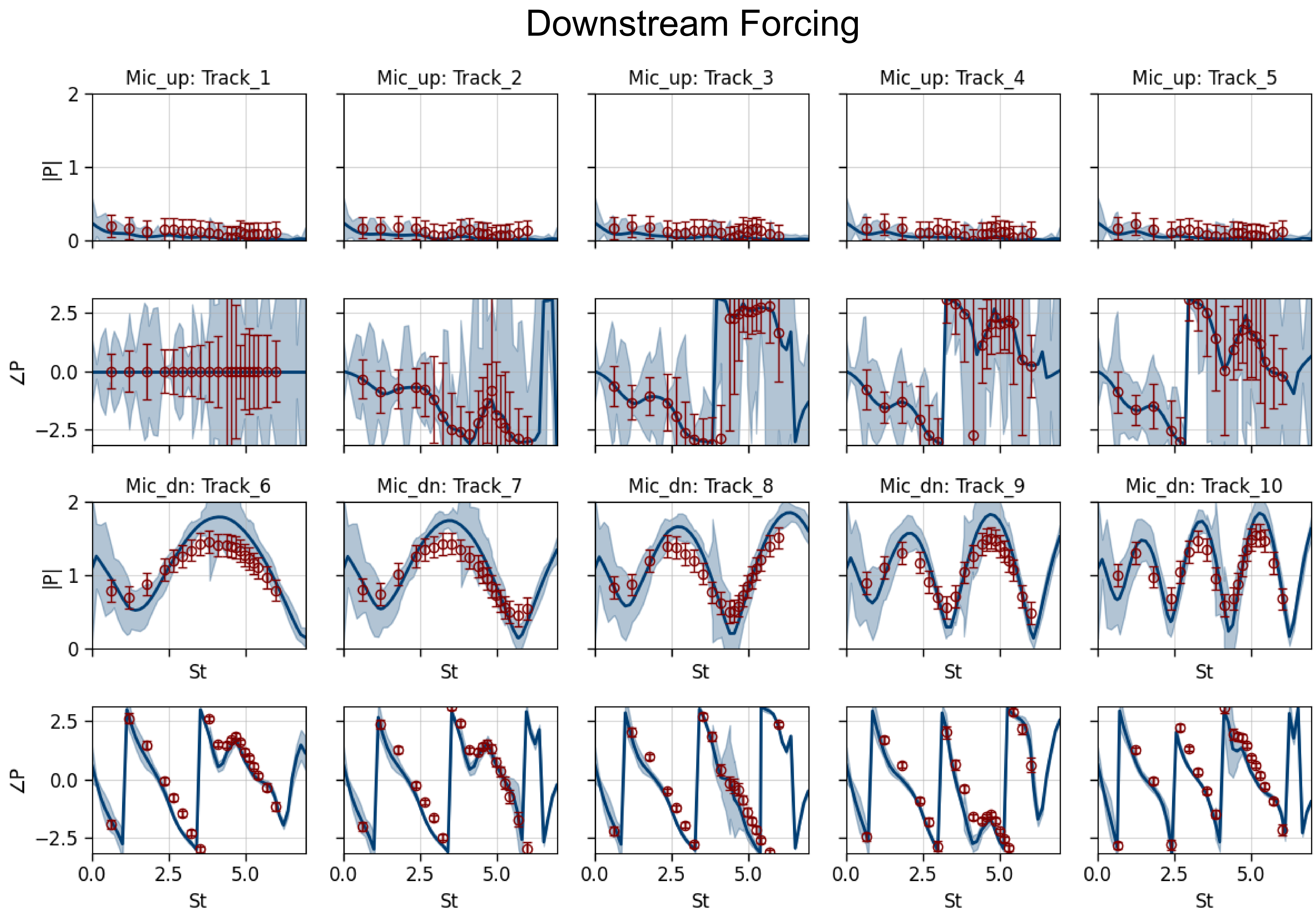

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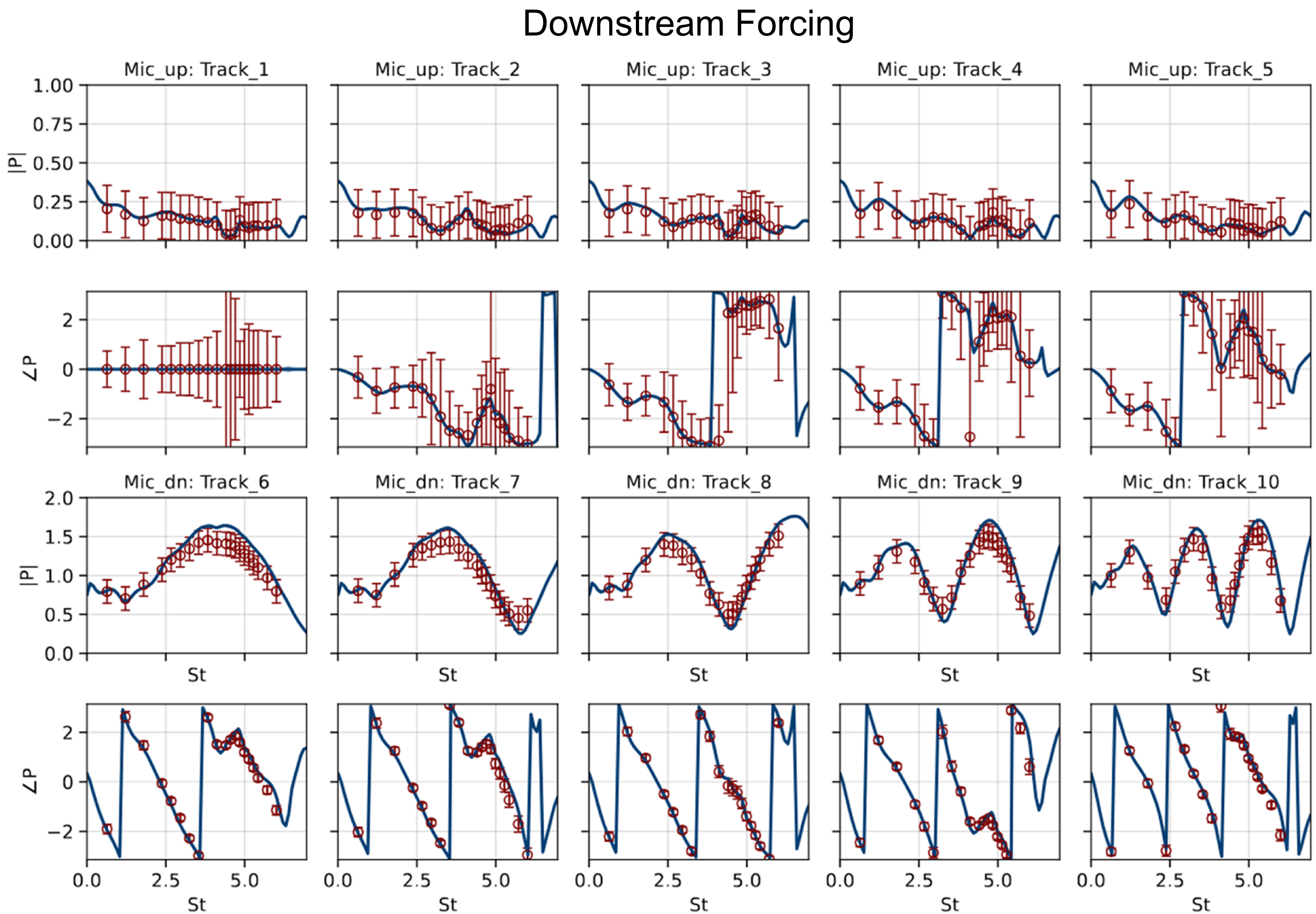
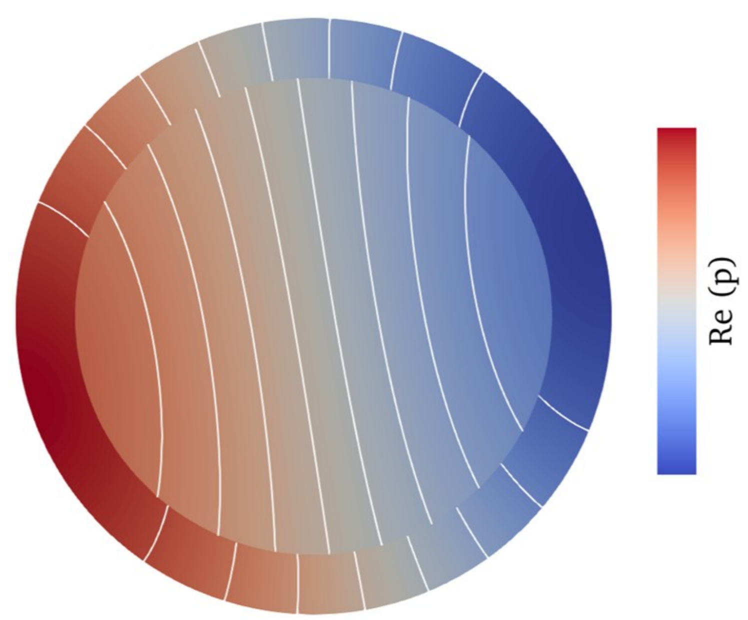


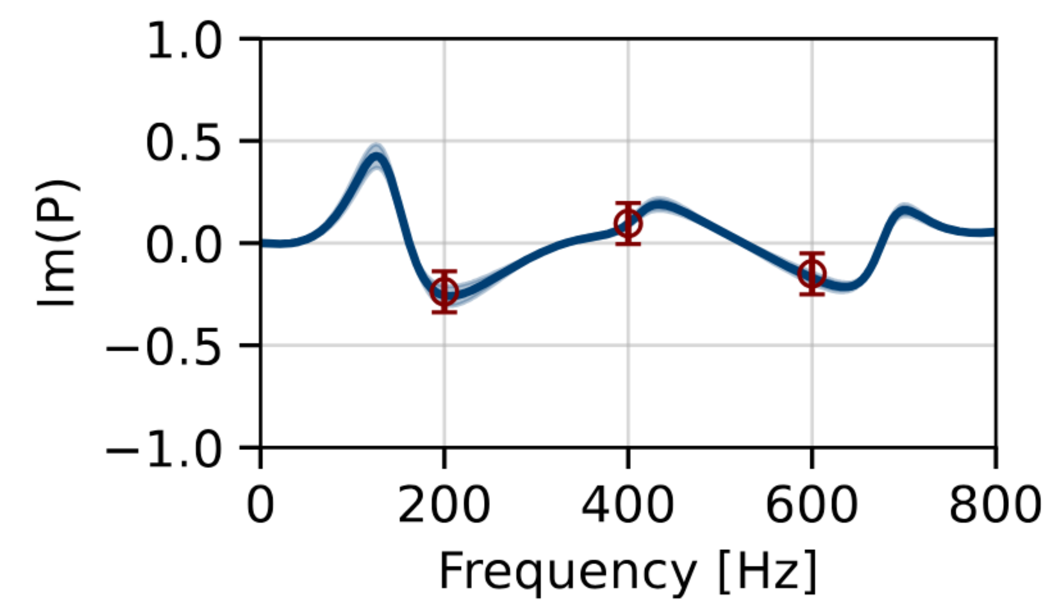
Helmholtz solver + Physics-based Models + adj-acc. Bayesian inference
➔ Quantitatively accurate, efficient 3D thermoacoustic models

Qualitatively accurate model | ***Quantitatively*** accurate model

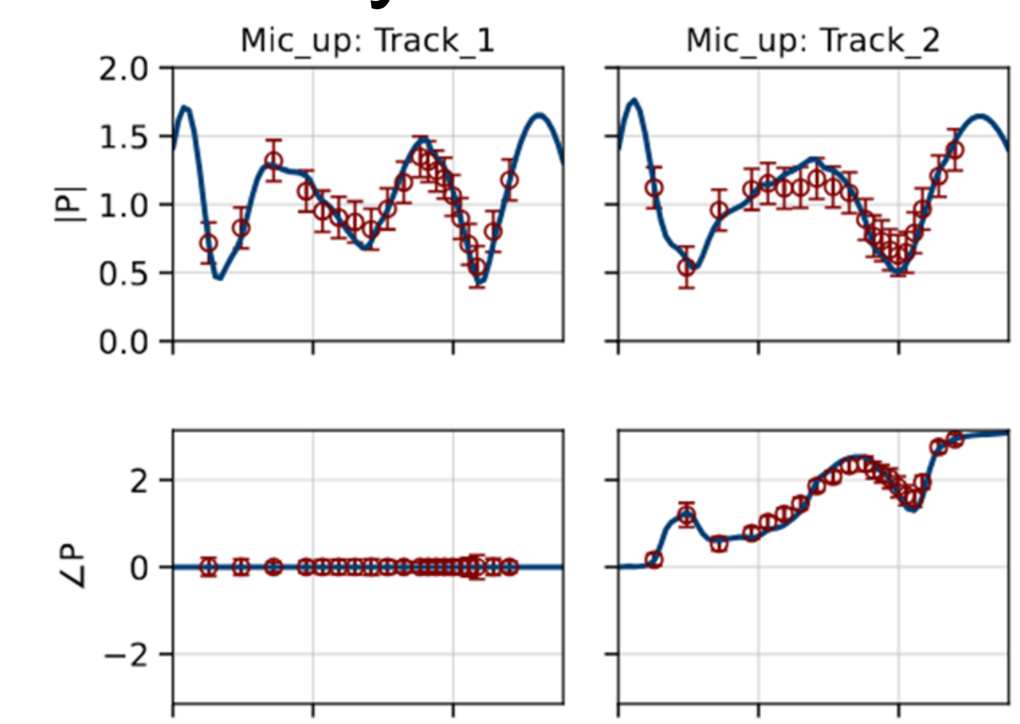
Acoustic Damping: Physcis-based models



Bayesian Inference: Learning from Data



Application: Rolls-Royce SCARLET rig



Mitigation of thermoacoustic instabilities

- More reliable shape optimization
- Physically interpretable parameters
- Interpolation with uncertainty

Future Work

- Reactive Conditions
- Extrapolation to unseen operating points

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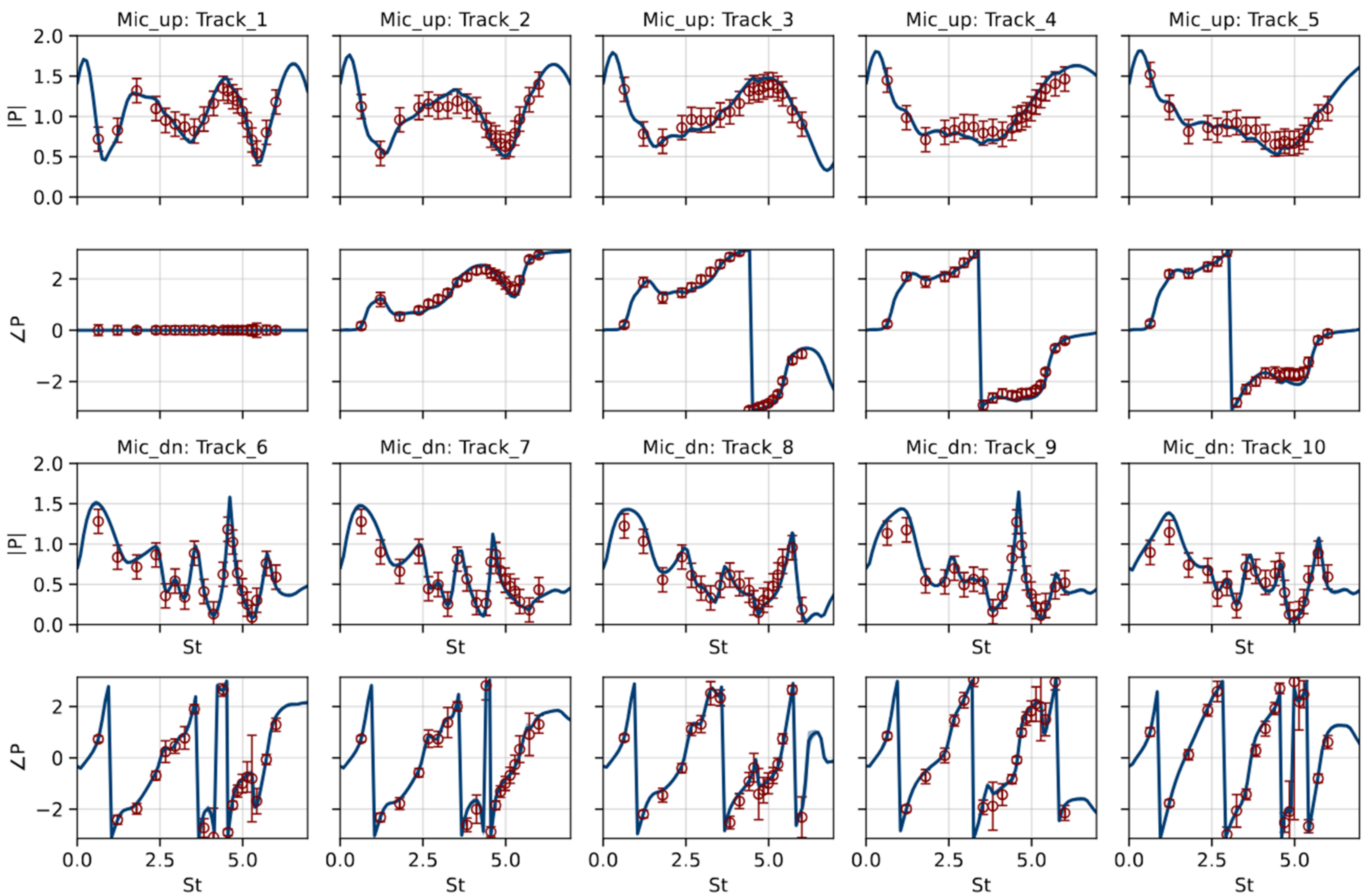
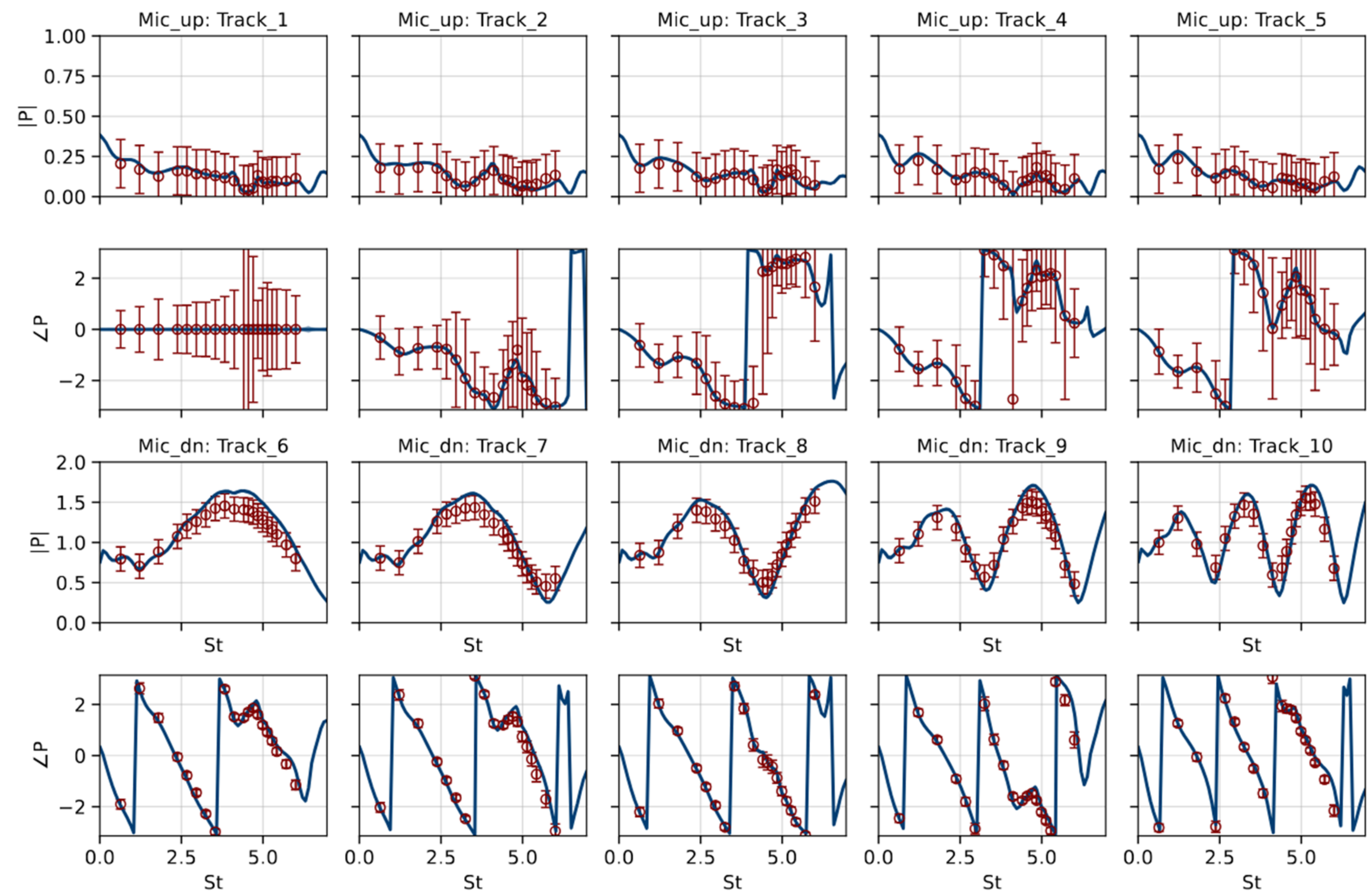
[50] Bruno B. H. Schuermans, Wolfgang Polifke, and Christian Oliver Paschereit. Modeling Transfer Matrices of Premixed Flames and Comparison With Experimental Results. In *Proceedings of the ASME Turbo Expo 1999: Power for Land, Sea, and Air*, volume Volume 2: Coal, Biomass and Alternative Fuels; Combustion and Fuels; Oil and Gas Applications; Cycle Innovations, page V002T02A024, Indianapolis, Indiana, USA, June 1999. ASME.

Data is assimilated to RR-SCARLET (20 Frequencies, 10 Microphones)

Upstream Forcing

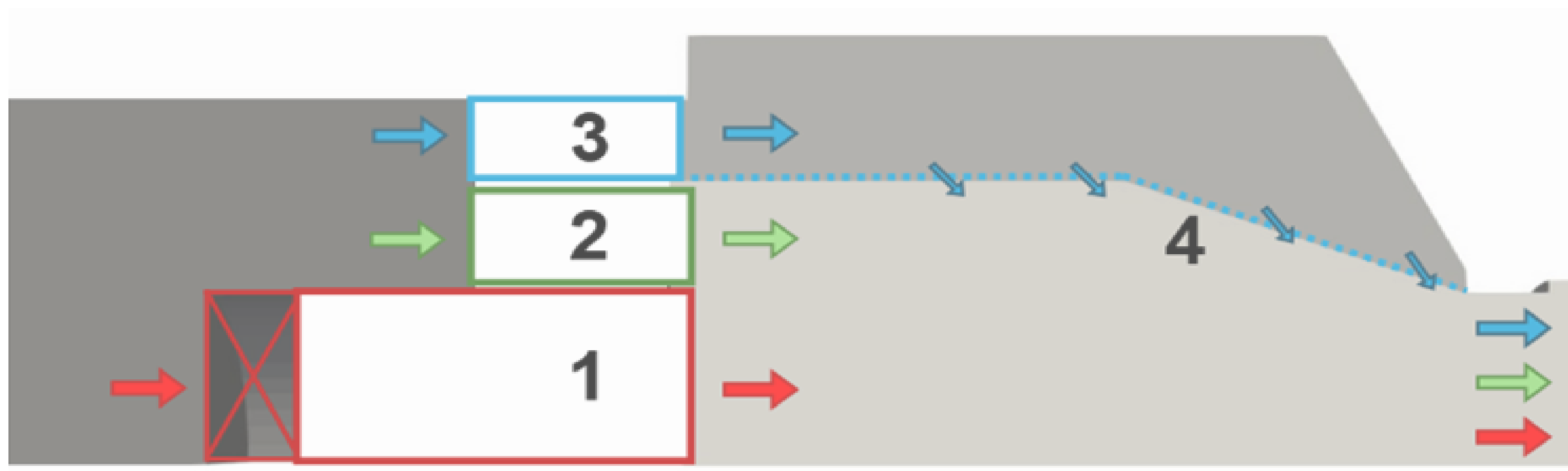
Downstream Forcing





— Model ($\pm 2\sigma$)   o Data ($\pm 2\sigma$)

The posterior joint marginal PDFs collapse to small regions in the parameter space indicating high certainty in the parameters



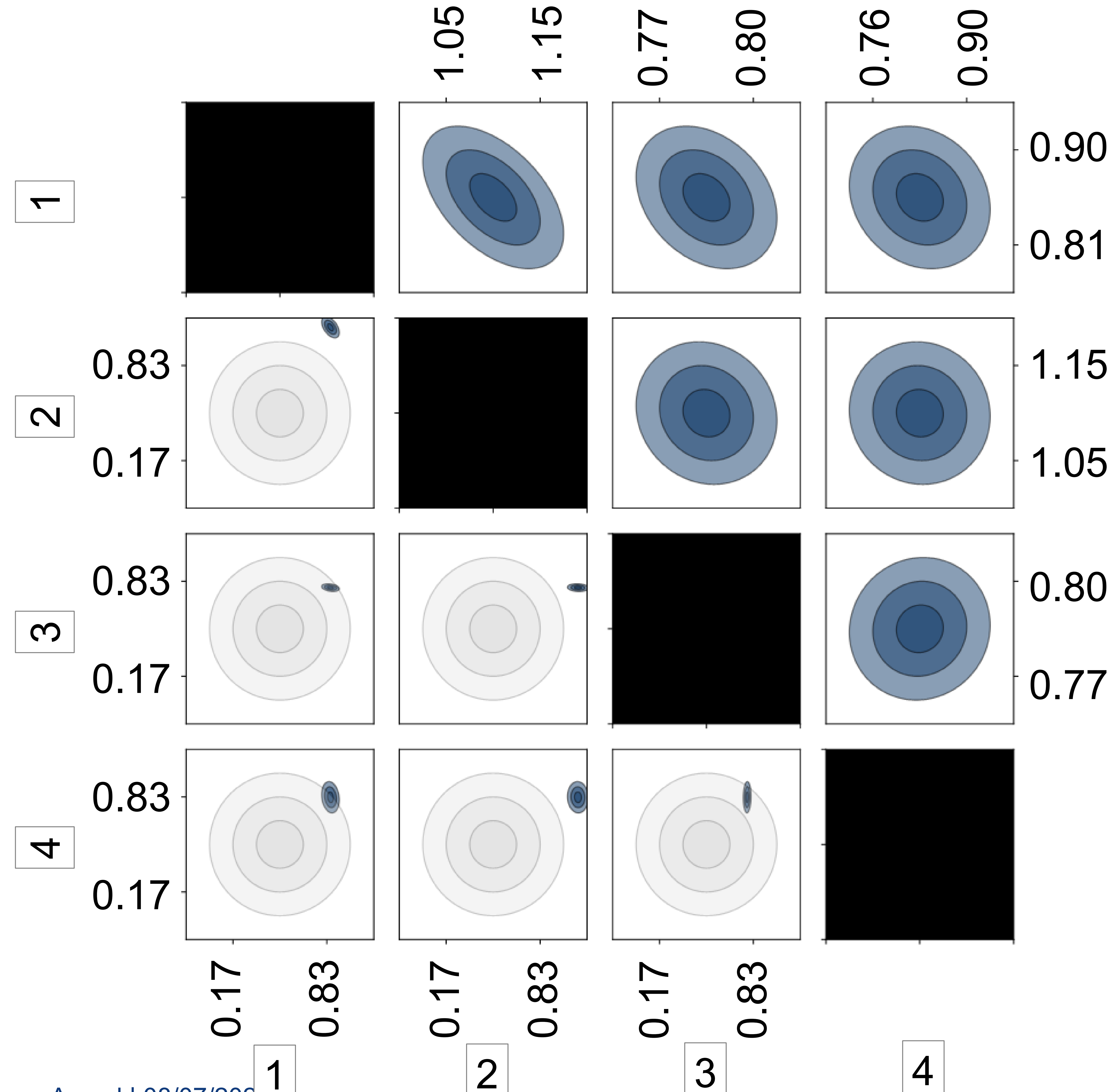
Effective area $\boxed{1}\ \boxed{2}\ \boxed{3}$

Discharge Coefficient $\boxed{4}$

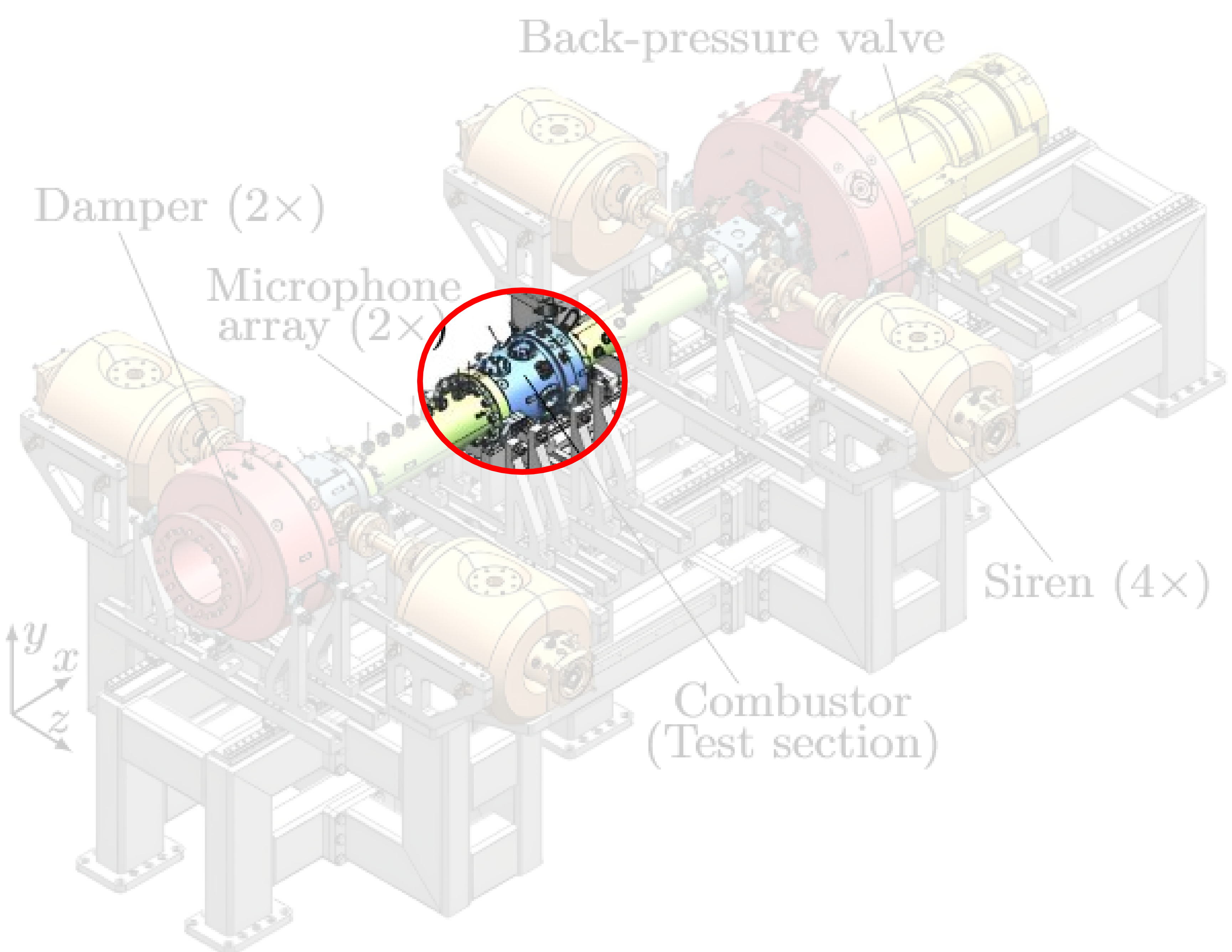
Joint Marginal PDFs:

a_{MP} : $\pm 3\sigma$, posterior PDF

a_0 : $\pm 3\sigma$, prior PDF

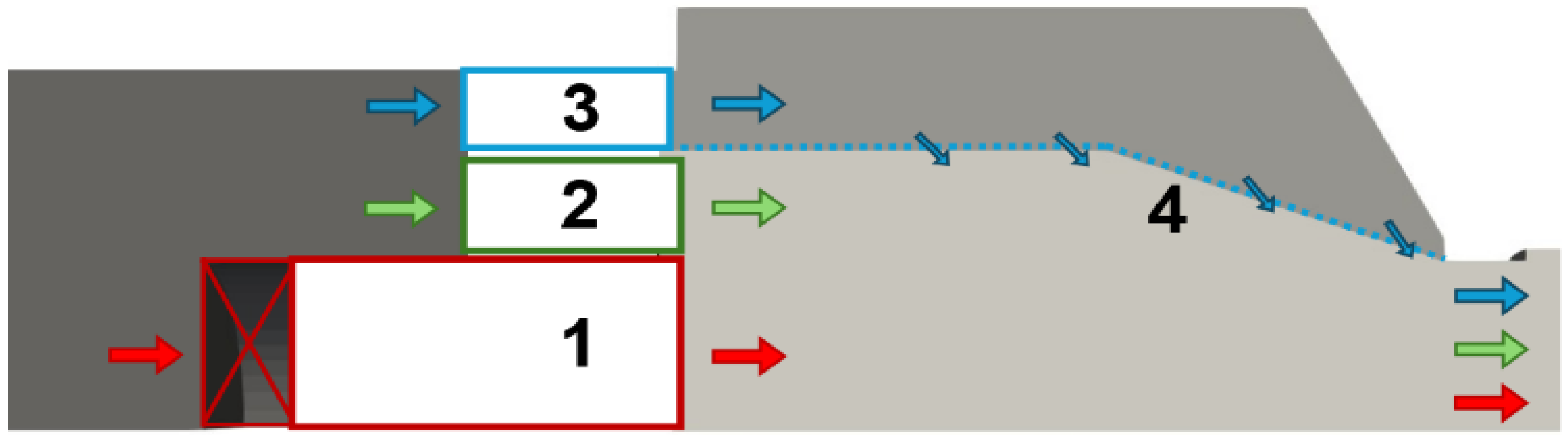


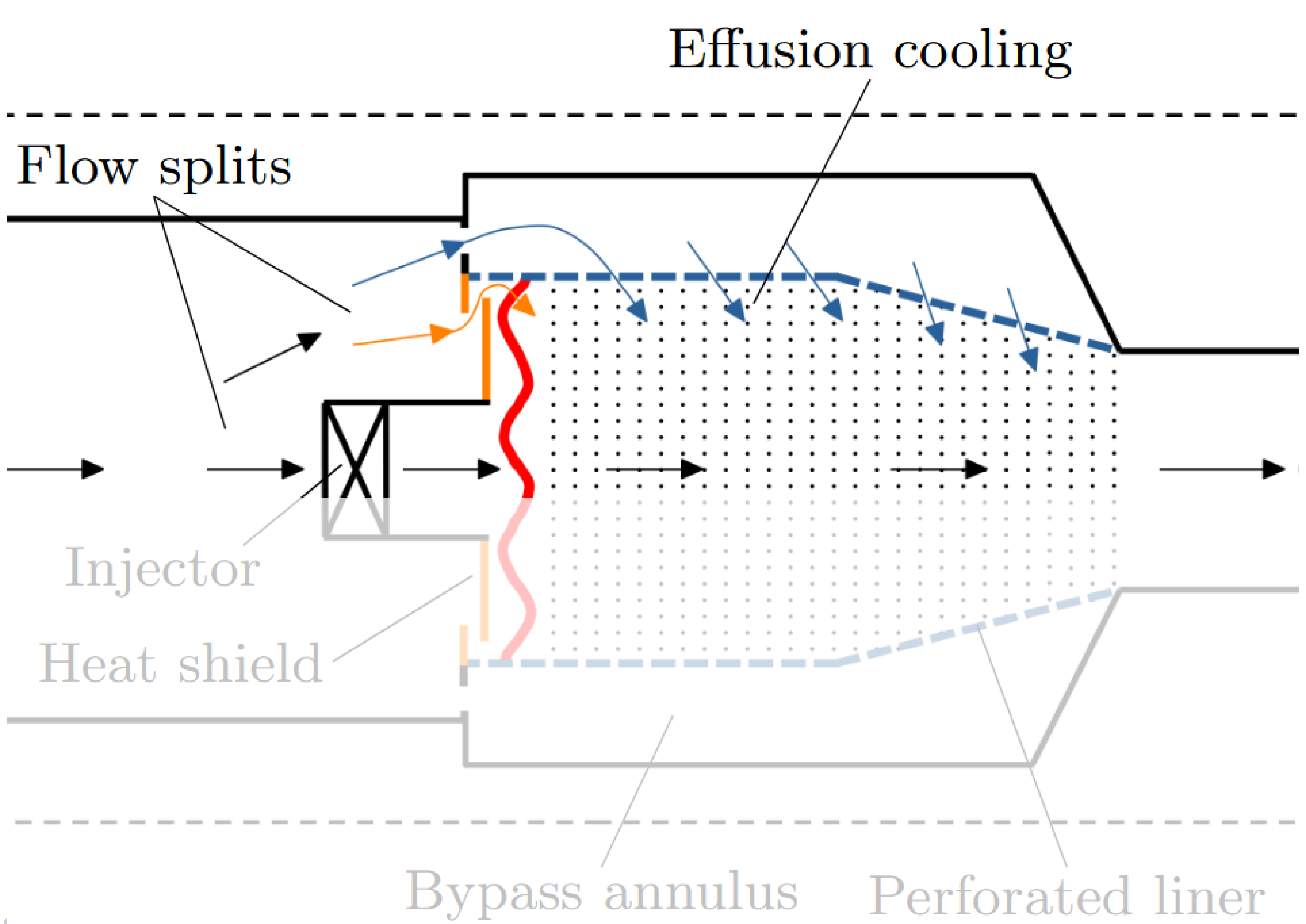
The combustion chamber in RR-SCARLET (25 bar, 950K, 4kg/s) comprises **complex flow features**



Eder et al., 2024

Helmholtz-solver
➔Physics-based models in **1, 2, 3, 4**





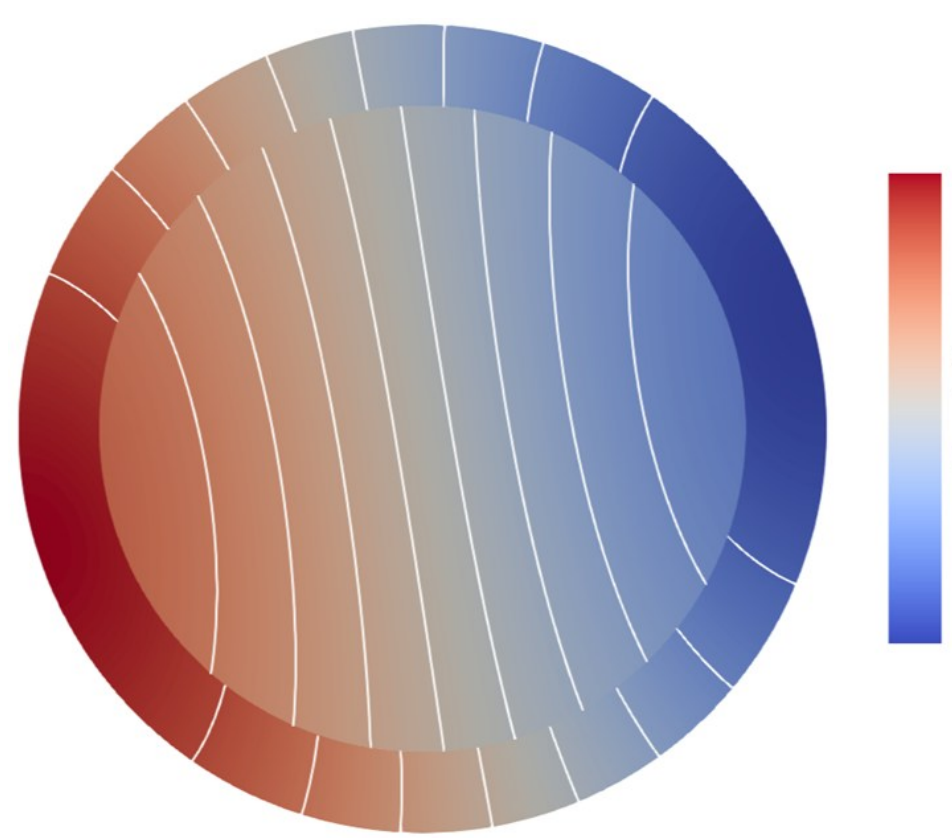
Can we create a **quantitatively** correct Helmholtz-Solver if we incorporate **physics-based models** and learn from **experimental data**?

Qualitatively accurate model | ***Quantitatively*** accurate model

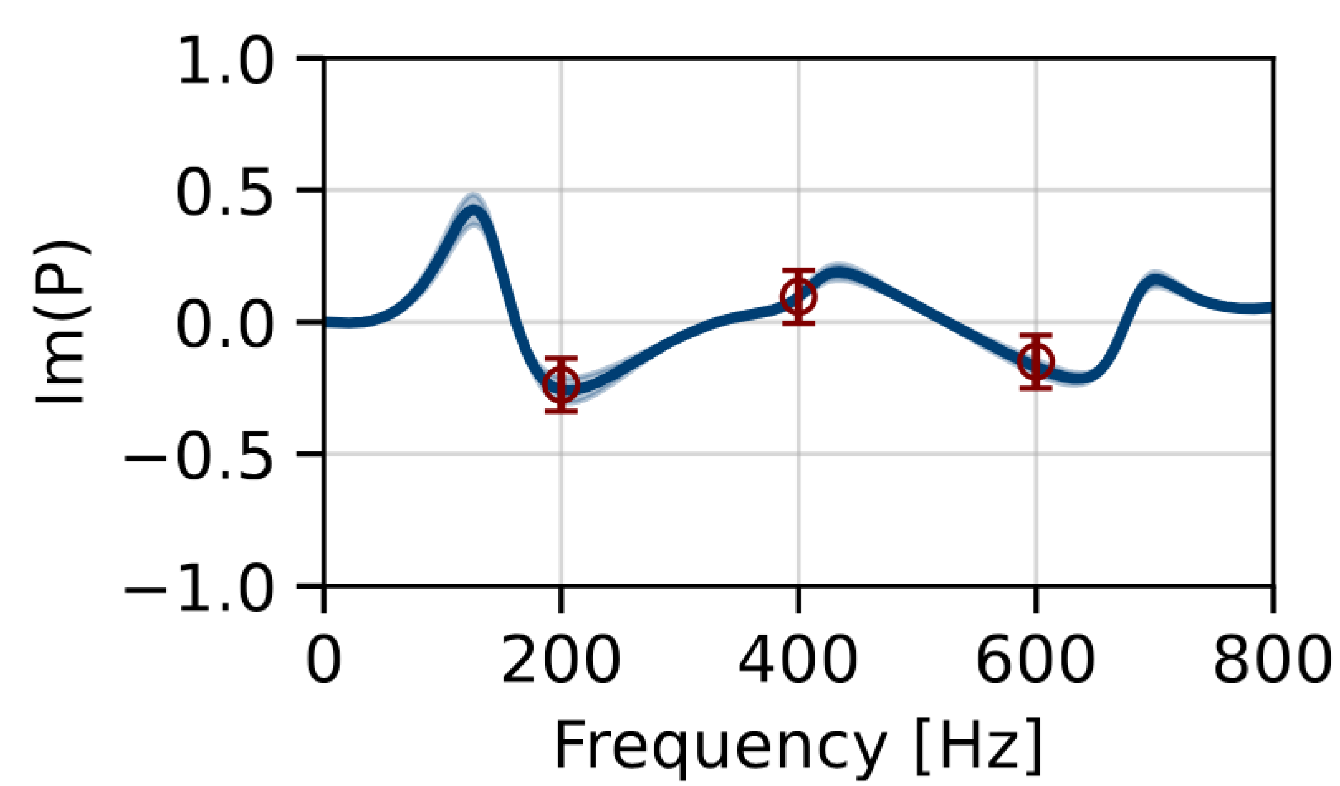
Methods

Results

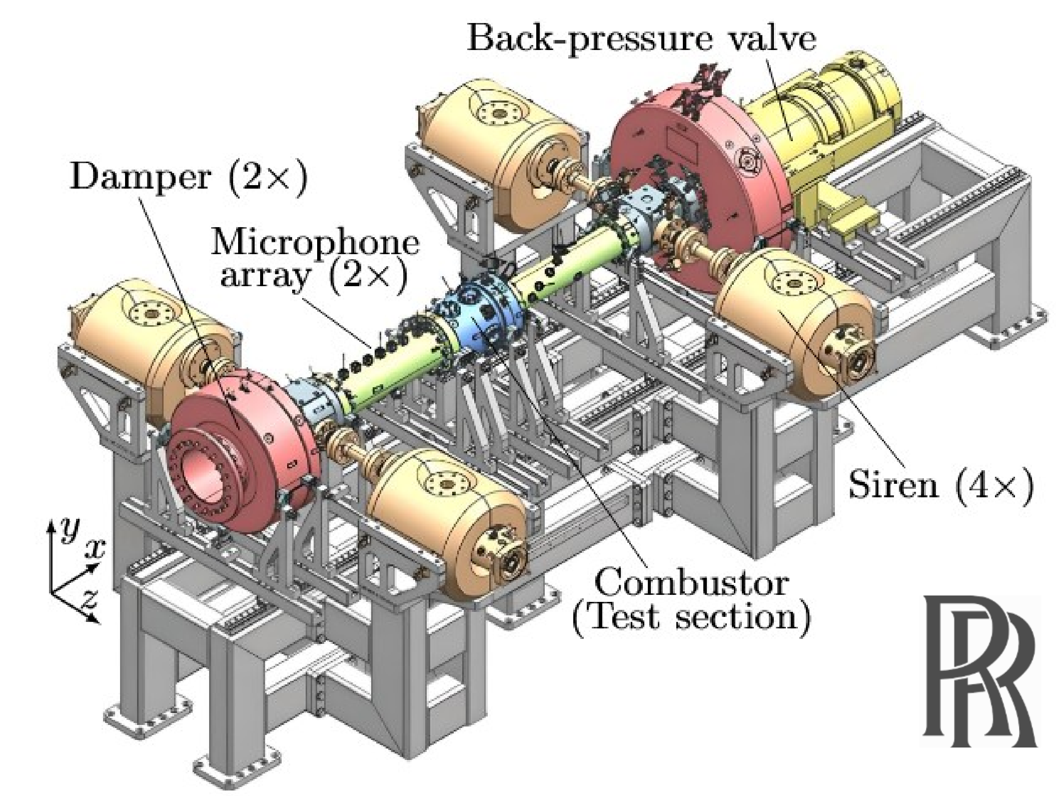
Acoustic Damping: Physcis-based models



Bayesian Inference: Learning from Data



Application: Rolls-Royce SCARLET rig



➔ *Novelty in the literature*

Data is assimilated to RR-SCARLET (20 Frequencies, 10 Microphones)