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GAS TURBINE UNCERTAINTY USING BAYESIAN STATISTICS

David Munn[1], Henry Catchcart[1], Daniel Guymer[1], Richard Green[2]

[1] Frazer-Nash Consultancy, Stonebridge Hours, Dorking Business Park, UK
+441306885050
j.douglas@fnc.co.uk
[2] Solar Turbines, 2200 Pacific Highway, San Diego, CA 92101, USA
+1619
Green_richard_j@solarturbines.com

ABSTRACT

Condition-based lifing methods have become more common for industrial gas turbines. Component temperature distributions are one key input into these advanced lifing methods, with analytical models used to predict these temperatures. These typically predict bounding temperatures, which do not accurately represent the variability in temperature that occurs in a turbine. As a result, there remains the potential for significant errors in predictions of component temperatures and significant pessimism in lifing assessments. The analytical models rely on a series of assumptions and are typically determined using the experience of thermal engineers and refined using experimental data. This process is time consuming, subjective, and does not give any indication of the potential error in the predictions.

This paper presents an approach for systematically combining the judgement of experts with measurements. Prior distributions for each model boundary condition are determined using engineering judgement, and then refined using Bayesian Inference from the available measurements. This approach leads to a map of most likely temperature, uncertainty and variability across the component. The technique can potentially expedite the process of thermal model matching, make it repeatable, provide distributions to probabilistic structural calculations, indicate the ideal locations for additional measurements and predict the value of measurement campaigns.

NOMENCLATURE

$p(x)$	Probability density function of a continuous random variable X taking a value x
$p(y\|x)$	Probability density function of a continuous random variable Y taking a value y, given variable X takes a value x.
x_i	Numeric value associated with the i^{th} boundary condition of a thermal model, such as temperature or heat transfer coefficient.
y_j	j^{th} Measured value during experimental study.

INTRODUCTION

Gas turbine components are historically designed based on an idealised operating cycle for the package, leading to a total number of operating hours, or maintenance cycles, that each component can withstand before replacement. However, the true operation of gas turbines can vary greatly depending on application, from continual part load operation, to frequent start-stop cycles. .

As a result of this variation, the design cycle has to be conservative in order to bound all possible operation histories. The level of this conservatism is unknown at the point of manufacture, which leads to the potential that components are replaced when they could have safely operated for a number of further maintenance cycles.

Condition based lifing assessments rely on real operating data from assets to predict the accumulated damaged and remaining useful life of individual components. Such condition-based assessments are gaining popularity as they can allow the lives of high-value components to be extended.

Two key damage mechanisms for main hot gas path gas turbine components are creep-fatigue interaction (Green et al 2016, and Green et al 2019) and high temperature oxidation as these components typically spend the majority of their operating life at high temperature and under significant centripetal stresses. Prediction of each of these damage mechanisms require a good level of understanding of the thermal environment experienced by the component.

Condition based assessments are reliant on structural and thermal analytical models. The construction and tuning of these models can be expensive, time consuming, non-repeatable and reliant on the judgement and experience of thermal and structural engineers. This paper aims to present an objective, repeatable method of model tuning, taking both measurement data and engineering judgement into account.

Thermal models are generally created and compared to available test data, either at the peak steady-state condition, or transiently if data is available. The model boundary conditions are then modified using the judgement of an experienced thermal analyst in order to achieve a close match to available test data. This comparison is performed at discrete locations where test data is available, so there is potential that there is a spatial variation that is not captured by the model. There is also a level of uncertainty around the test data as the operating conditions may vary, or other potential issues associated with measurement campaigns.

These effects combine meaning that there is a large and unknown level of uncertainty over the thermal prediction, and hence conservatism must be applied. Steps are taken to reduce this uncertainty, such as running multiple measurement campaigns or various thermal models. This paper will introduce a framework for the efficient and objective evaluation of these uncertainties.

As this approach combines the uncertainty of the model and the uncertainty of the test data it can be used to determine the value of work, whether that is additional measurements or further modelling effort. The approach becomes even more powerful when combined with condition monitoring, as the continual gathering of real data can be used to update the models.

APPROACH

A thermal model of a gas turbine blade generally contains a number of boundary conditions, which are determined using various sources. Sources of boundary condition information include, but are not limited to:

- 1D flow network solvers;
- Computational fluid dynamics;
- Empirical heat transfer correlations;
- Engine performance models.

Boundary conditions applied to the model include convective heat leads, windage, fluid mass flow, heat fluxes and others. Each boundary condition is applied transiently to the model either as a uniform value or as spatially varying.

For example, the 1D flow network solver can be used to determine secondary cooling mass flowrates and convective heat transfer coefficients. This requires the flow path to be simplified to a series of connectors with geometrical approximations applied to each connector. Empirical correlations are then used to calculate pressure drop and mass flow rates. Heat transfer coefficients require an accurate estimate of the wall temperature.

Each of these sources, and therefore applied boundary conditions, will have an uncertainty distribution associated with it. Current methodologies do not consider these underlying uncertainties, and instead boundary conditions are manipulated and modified to achieve a match to available test data. The level of modification can vary from simply factoring heat transfer coefficients to running flow network solvers at different conditions. This process of manipulation ignores any remaining uncertainty due to either the boundary conditions or the measurements and requires significant effort from experienced engineers.

Typically this effort is replicated across multiple components, which can cause conflicts between the component and system level manipulations. If considered as a system then the level of match on each component may be poorer. If each component is considered independently then the combination of predictions could be unphysical.

The approach presented in this paper aims to perform a probabilistic calculation to determine the likely values of the boundary conditions and the uncertainty in temperature across the component, taking any available measurements into account.

The approach begins by defining initial probability distributions for all of the model boundary conditions, $p(x)$. These distributions are termed the "priors" and capture engineers' knowledge about physically reasonable values for the boundary conditions.

The priors are then refined by considering the available test data using Bayes theorem (Equation 1). This equation calculates a distribution of the boundary conditions given the measurements, $p(x|y)$, termed the posterior. As additional measurements become available this process can be repeated to incrementally refine the distributions of the boundary conditions.

The third term in Bayes theorem, $p(y|x)$ is termed the likelihood and is defined as the probability of the measurements for a given set of boundary conditions. The final term, $p(y)$, can be expressed as an integral over values of x (Equation 2) and hence becomes a normalising constant.

$$p(x|y) = \frac{p(y|x)p(x)}{p(y)} \tag{1}$$

$$p(y) = \int p(y|x)p(x)dx \tag{2}$$

To calculate the likelihood for a set of boundary conditions, x, the temperature at the measurement location must be predicted and compared to the measurement and its associated uncertainty. Characterisation of the posterior distribution requires this calculation to be performed a large number of times, which is impractical with the complexity of most thermal models (Zentuti et al 2017).

A response surface is therefore developed for the model to ease computational burden. Response surface methods aim to fit a simplified model to the results of a set of full assessments and then use a simplified model to predict the results for a sampled set of inputs (Cathcart et al 2019).

In order to generate this response surface, the analytical model must be run for varying input boundary conditions. For a highly complex thermal model like a bladed-disk assembly the set of inputs requires careful consideration, performing perturbations on groups of applied heat transfer loadings in a coherent manner. For example, if a secondary cooling mass flow is changed then the convective heat transfer correlation should also be changed by an equivalent amount. The thermal model is solved at steady-state conditions for a large number of cases, and temperature predictions exported for each.

A response surface, or emulator, is generated from these sensitivity runs by determining the partial derivatives of the temperatures with respect to each boundary condition. Although the response surface is significantly less complex than the full simulation it is still infeasible to calculate an analytical expression for the likelihood or posterior distributions and hence Monte Carlo methods are used to describe the posterior. As the emulator only allows point-evaluation of the likelihood and hence posterior probability density functions Markov Chain Monte Carlo (MCMC) method is used to sample all of the probability distribution functions. The random walk Metropolis-Hastings algorithm is used, which samples and evaluates points within the distributions, and walks through the distributions, prioritising moves towards more probable data. The application of the Metropolis-Hastings algorithm to the prior function is trivial, as the central estimate prediction can be used as a sensible starting point. However, in order to apply the algorithm to the likelihood and the posterior a simple optimization algorithm is first used to find a good starting point for the sampling process. A visualisation of the random walk optimisation is shown in Figure 1.

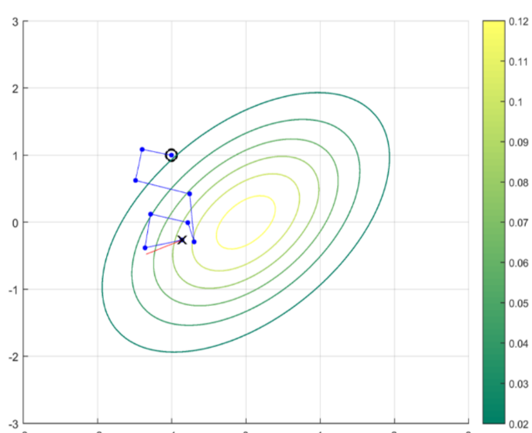

FIGURE 1: VISUALISATION OF METROPOLIS-HASTINGS RANDOM WALK ALGORITHM

If the starting point selected for the posterior or likelihood is poor, the initial samples may not be representative of the desired distribution. Therefore, a burn-in period consisting of the first 10% of sampled points is disregarded. A typical example of the negative log-likelihood of during the burn-in and sampling periods is shown in Figure 2.

An example set of prior, likelihood and posterior distributions produced by the process for a single boundary condition can be seen in Figure 3.

Having characterised the posterior distribution the response surfaces can be queried to provide the temperature distribution any location in the model. This information can be used to identify regions of high uncertainty which could be targeted for further measurement studies. These temperature distributions can also be supplied to stress analyses in order to refine stress and life predictions. The Bayesian approach can be applied to these structural models, with the probability distribution of the thermal input defined by the method described here.

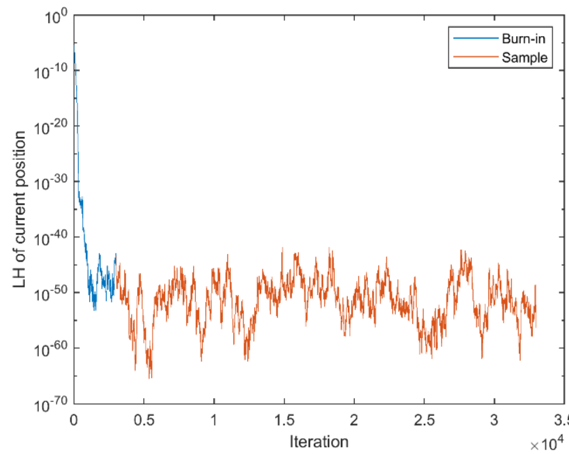

FIGURE 2: EXAMPLE BURN-IN AND SAMPLING PERIOD

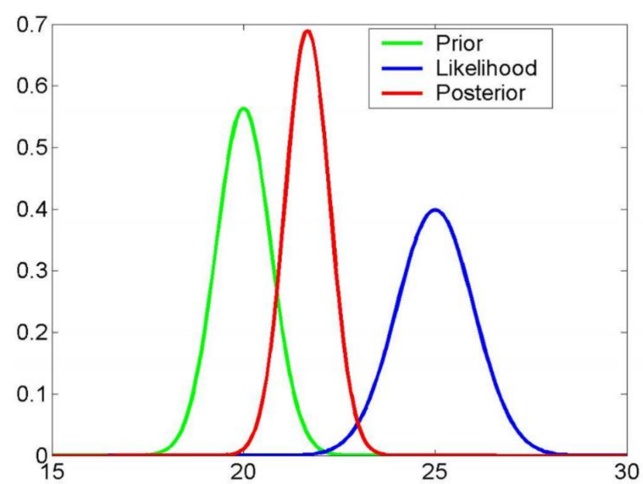

FIGURE 3: EXAMPLE PRIOR, LIKELIHOOD AND POSTERIOR DISTRIBUTION FOR A SINGLE BOUNDARY CONDITION

RESULTS

An example thermal model of a generic high pressure stage one blade was developed in ANSYS APDL. This relatively complex model consists of over two hundred separate boundary condition regions. A convective load applied to the aerofoil which is defined with a fixed fluid temperature and spatially varying heat transfer coefficient. Blade internal cooling was modelled using fluid elements with specified mass flow, which are linked to the internal surfaces with applied heat transfer coefficients. A number of other convective and conductive boundary conditions are also applied.

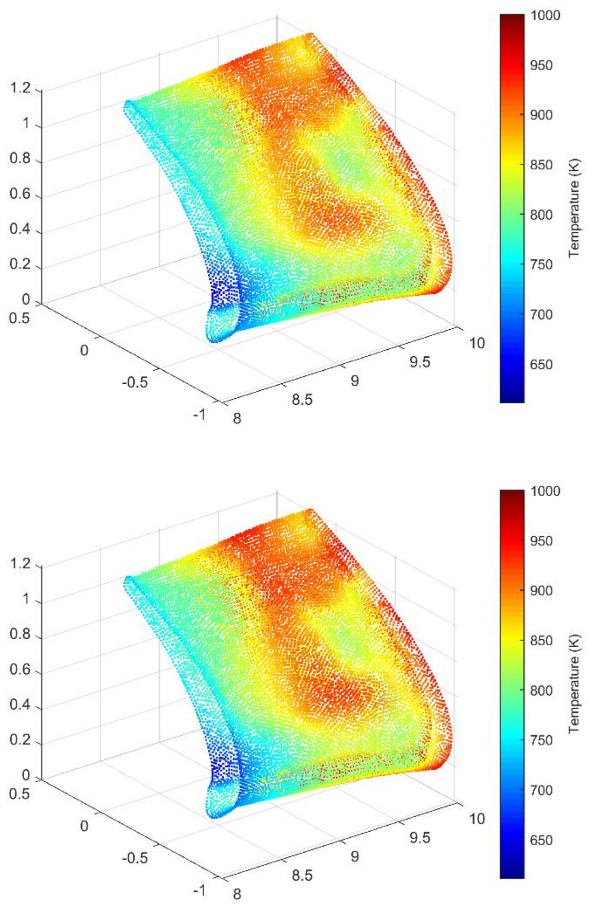

FIGURE 4: CENTRAL ESTIMATE (UPPER) AND PERTURBED (LOWER) NODAL TEMPERATURE PREDICTIONS FROM ANALYTICAL MODEL

A percentage perturbation is defined for each boundary condition, or group of boundary conditions. The amount of perturbation is based on engineering judgement. The model is then run for a central estimate case, and an additional case for each boundary perturbed once. This results in over two hundred steady-state solutions which are used to generate the response surface, assuming a linear response of nodal temperature result. An example of the central estimate and perturbed nodal temperature solutions are shown in Figure 4. The central estimate is identical to the prior sampled at the 50.0th percentile.

The standard deviation of the central estimate is shown in Figure 5.

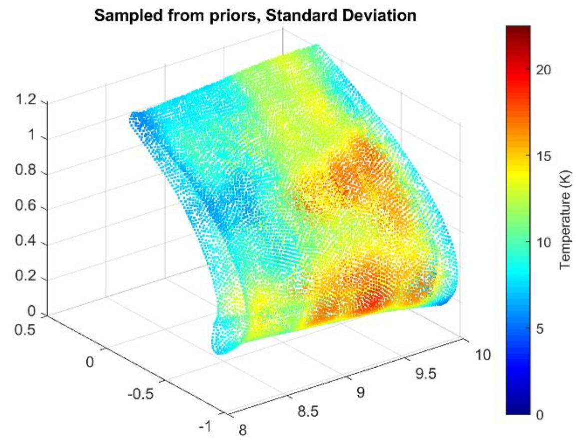

FIGURE 5: STANDARD DEVIATON OF PRIOR

Example test data was created for 27 locations as shown in Figure 6. This presents a realistic number and spread of thermal crystal locations which are commonly used to capture peak temperature from an engine test. Intentionally, there are fewer crystals at the tip of the blade. This presents as a larger standard deviation in likelihood. A standard deviation at each measurement location of 3.5 K is used with a normal distribution to define the likelihood. The resultant central estimate and standard deviation of the likelihood is shown in Figure 7.

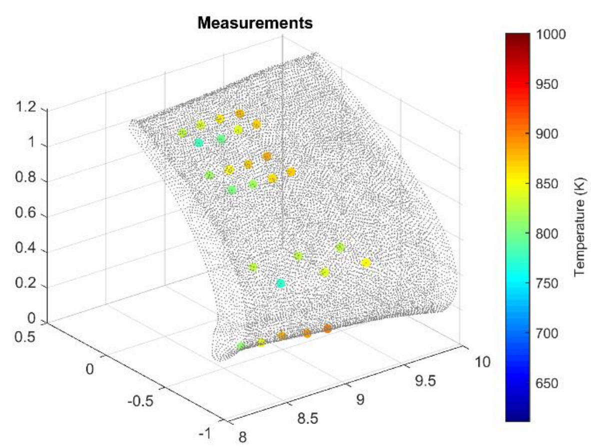

FIGURE 6: MEASUREMENT LOCATIONS

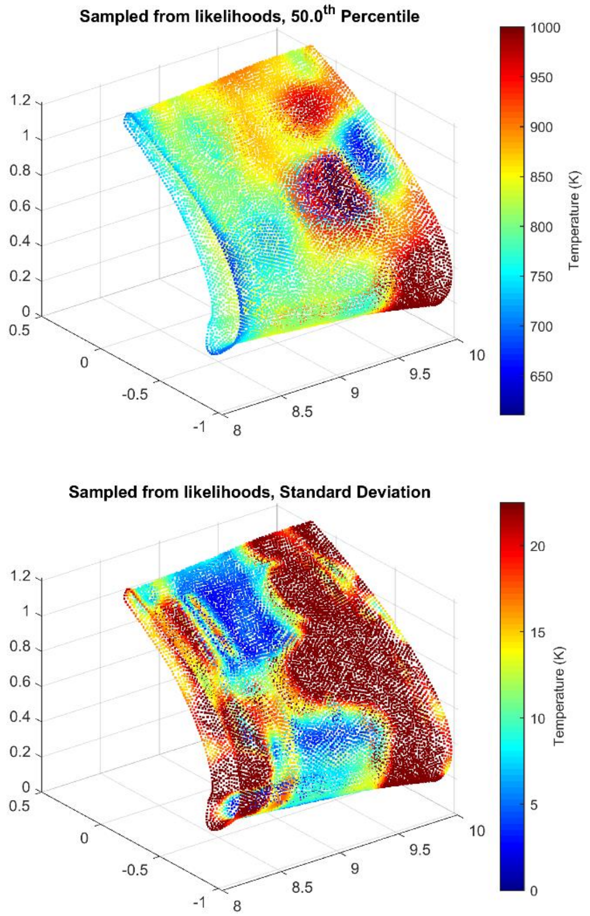

FIGURE 7: CENTRAL ESTIMATE (UPPER) AND STANDARD DEVIATION (LOWER) OF LIKELIHOOD.

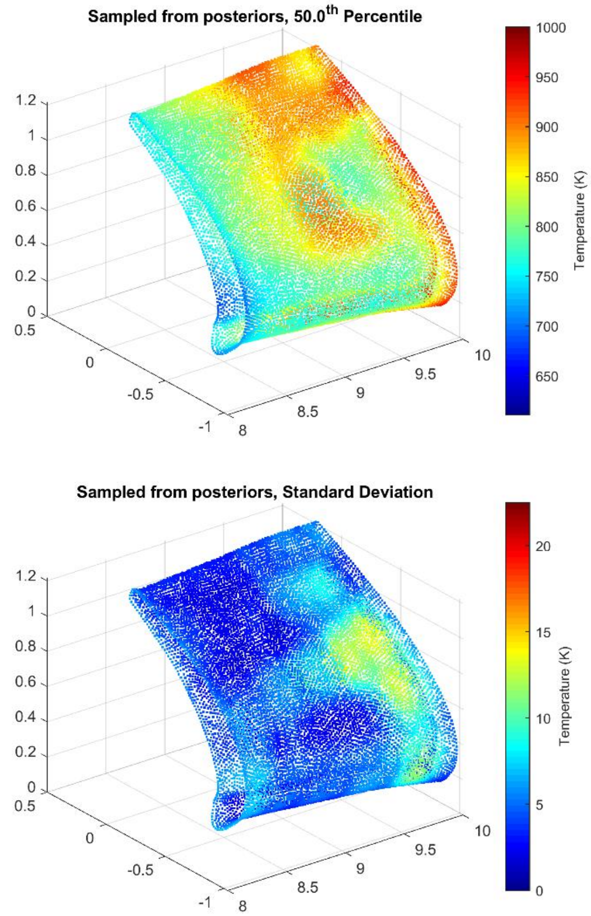

FIGURE 8: CENTRAL ESTIMATE (UPPER) AND STANDARD DEVIATION (LOWER) OF POSTERIOR.

The response surface is generated and the MCMC algorithm is used to generate the central estimate and standard deviation of the posterior as shown in Figure 8.

The three response surfaces are sampled at the measurement locations, comparing the distribution from the three functions. Examples at two locations are shown in Figure 9.

DISCUSSION

The developed framework allows a manufacturer to gain a better understanding of the uncertainty in the thermal predictions of their components, and hence life and maintenance schedules. This should result in financial benefits for the manufacturer or operator of equipment.

As the outlined approach develops a spatial map of uncertainty, this can be used to specify further testing and evaluate value for money on proposed tests. The value returned by a test can be simulated by running the Bayesian analysis with additional points, updating the likelihood response surface.

Bayes' theorem allows for continuous revision of the posterior using new measurements and setting the previous posterior to be the new prior. This approach is ideal for condition based lifing assessments, which rely on accurate reduced order models. Reduced uncertainty in life predictions allows for greater benefit to be gained from the condition based monitoring approach, resulting in greater component re-use.

The proposed method requires the thermal analyst the set reasonable perturbations for each boundary condition. Unrealistic precision applied to the probability of boundary conditions will propagate through the analysis to the posterior. Similarly, boundary conditions must be specified in a reasonable manner or with sensible grouping. If this is not done then the proposed method would produce a set of unphysical boundary conditions.

Existing tool sets may need to be updated to manage the required simulations. The size of the mesh also becomes significant, as the nodal results are exported for each solution. The approach presented has been demonstrated for a steady-state solution, but comparison to transient thermocouple data is also available, but will require more processing time.

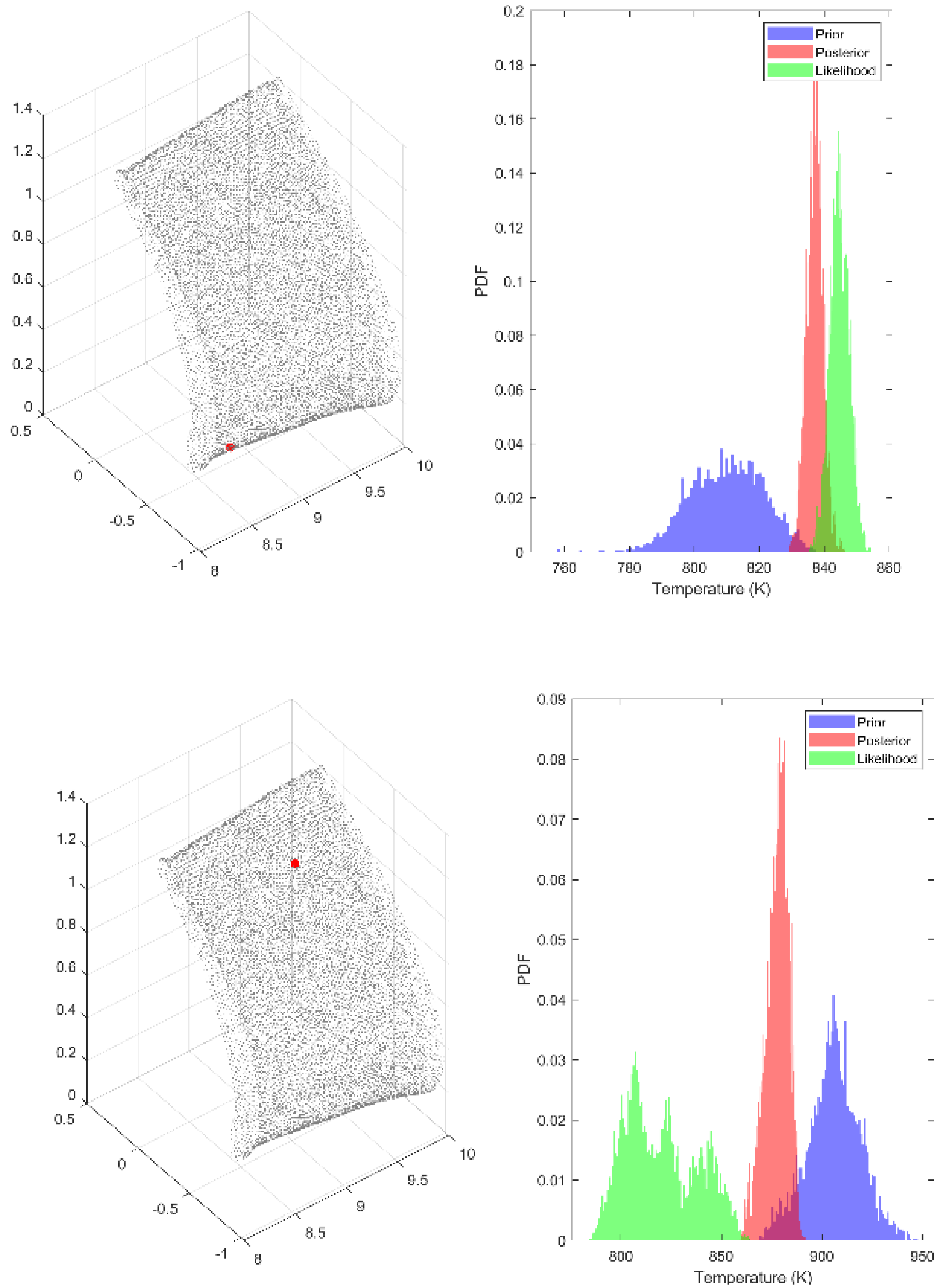

FIGURE 9: TWO EXAMPLE DATA LOCATIONS AND RESULTING PRIOR, POSTERIOR, AND LIKELIHOOD DISTRIBUTIONS.

ALTERNATIVE APPLICATIONS

This approach is generic and can be applied to a number of different applications. Any instance where an analytical model is created and compared to an alternative dataset. As already mentioned, structural models fit ideally into the same framework described here.

An example of an energy storage application has been developed to demonstrate this technology. The model is shown in Figure 10 below. The model has five convective heat transfer surfaces, one conductive boundary to the ground, and an internal heat generation. In order to build a response surface, the convective loadings and internal heat generation are varied one-factor-at-a-time (OFAT). The OFAT approach is reasonable for this test case as the run time is low. Other approaches can be used to generate the required response surface, for example multi factor variation. A total of 25 steady-state thermal simulations were run.

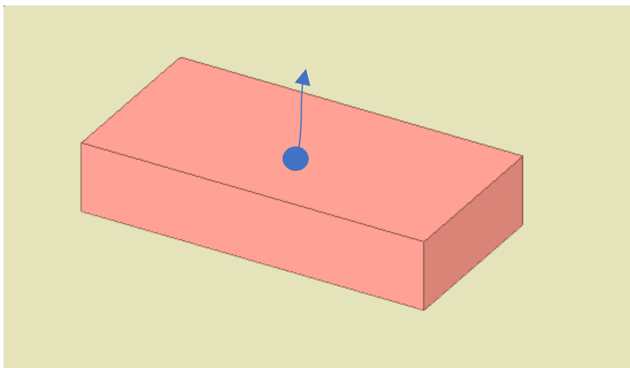

FIGURE 10: ENERGY STORE MODEL DOMAIN, WITH INTERNAL HEAT FLUX SHOWN WITH BLUE ARROW

Test data was manually generated for a single point, and this was input to the statistical analysis framework to derive the most likely set of boundary conditions, and an associated standard deviation of the posterior. The suggested boundary conditions were then input to the thermal model, which was re-run and produced the predicted comparison to the test data.

This example could then be developed further, for instance aiding on measurement campaign decisions, such as whether it is more beneficial to more accurately measure one additional location, or measure two locations with greater standard deviation. All additional measurement data would lead to a decrease in standard deviation of the overall thermal prediction if specified correctly.

In the example of an energy store, this would decrease the uncertainty of the total heat stored in the system, and would allow system level operation decision to be made regarding the use of the energy store.

An alternative but similar example would be a nuclear fuel storage facility, where spent fuel casks are stored, but continue to release heat energy at a low rate. The approach identified could help decisions regarding the level of ventilation required to keep the facility within a required temperature range.

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