DELIVERABLE 4.2.2

FP7-ENERGY-2008-TREN-1

H2IGCC Energy without CO_2

Grant Agreement No:	**239349**
ACRONYM:	**H2-IGCC**

Description of the Models adapted or developed *ad hoc* for the IGCC&CCS plants

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Date of issue 29-03-2012

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Nomenclature

ξ	Stagger Angle
β	Blade Angle
LE	Leading Edge
TE	Trailing Edge
w	Relative Velocity
δ	Deviation Angle
θ	Camber Angle
m	Mass; Mass Flow
l	Chord
s	Pitch
i	Incidence Angle
h	Enthalpy
p	Pressure
Δp	Pressure Drop
ρ	Density
A	Area
fb	Blockage Actuality Functions
L	Work
u	Velocity
T	Temperature
R	Gas Constant
c_p	Specific Heat
ω	Coefficient Loss
C	Coefficient
σ	Solidity
f	Deterioration Factors
ε	Flow Deflection; effectiveness
η	Efficiency
Γ	Fuel Low Heating Value
XM	Fraction Composition
α	Air Fuel Ratio
K	Factor
CRF	Combustion Chamber Correction Factor
ΔT	Temperature Difference
S	Heat Transfer Surface, Entropy
ψ	Velocity Loss Coefficient
Ma	Mach Number
A_g	Throat Section
μ	Dynamic Viscosity
P	Mechanical Power
Q	Heat Quantity; Thermal Power
U	Heat Transfer Coefficient
τe	Thermal Conductivity
ce	Heat Capacity
NTU	Number of Transfer Units
κ	Ratio
k	Heat Ratio
Z	Number of blades in a row

Subscripts

d	deflection
j	inlet, value order
r	rotor
E	Euler
DS	secondary losses
L	lift
ref	reference
DA	annulus losses
p	profile losses; pressure
f	fuel, loss
c	cooling
u	outlet
i	inlet
w	water
b	blade
s	sonic
cr	critical
g	gas

INDEX

o	total value
*	reference value

1. Introduction

In recent years, due to the growth of powerful computing resources, the development of robust computational models and efficient algorithms has led to develop detailed models able to describe the behaviour of single component and of complex plants.
Two different approaches can be followed:

- Component behaviour is described by Input-Output relationships that can be represented and implemented by global equations;

- Component behaviour is described by models simulating the physical behaviour of machines and apparatuses. Different goals can be taken into consideration:

  - preliminary thermodynamic evaluations of cycle processes by taking global parameters into account;

  - determination of sizes that represent the design of machines and apparatuses at the nominal conditions. Such models are oriented to machines and apparatuses performance calculations taking also governing equation source terms (work exchange, heat transfer, entropy production, species etc.) into consideration. Empirical relations based on manufacture and user data, technical literature background and so on, can be taken into account;

  - component and plant off design and part load behaviour analyses under steady-state and transient conditions can be calculated.

Nowadays, plant behaviour can be described by elementary modules representing single components or macro modules describing the behaviour of complex plant sections. To represent the plant such modules are matched together. Various platforms allow an easy build-up of the plant but the component models quite often can be adapted to new machines and apparatuses with some difficulties. To overcome this problem RO3, in connection with past EU Projects, has developed models that have shown to be valid and flexible.
Such RO3 models are easily adaptable to new components and can allow the introduction of modifications for machines and apparatuses.

In the following paragraphs a general mathematical representation of the plant is given together with the solution approach. Also the models are presented. Finally the component models adapted by RO3 Research Unit to establish a 300 MW F Class Gas Turbine simulator are shown.

2. Background

Each thermo-mechanical system usually is made of many components each of which is devoted to one process transfer of heat, of work, of combustion etc.. Closed loop plants or equivalent ones (like IGCC plants, steam cycles, gas turbines, combined cycles, etc.) can be plotted as in fig. 2.1 where blocks 1, 2 ... represent components or group of them. In such a figure connections between components, input and output streams are schematically represented (E_{In}^{U} being the vector of the useful inlet quantity fluxes, E_{O}^{U} being the vector of fluxes of useful quantities, while E_{O}^{R} being the vector of fluxes of rejected quantities).

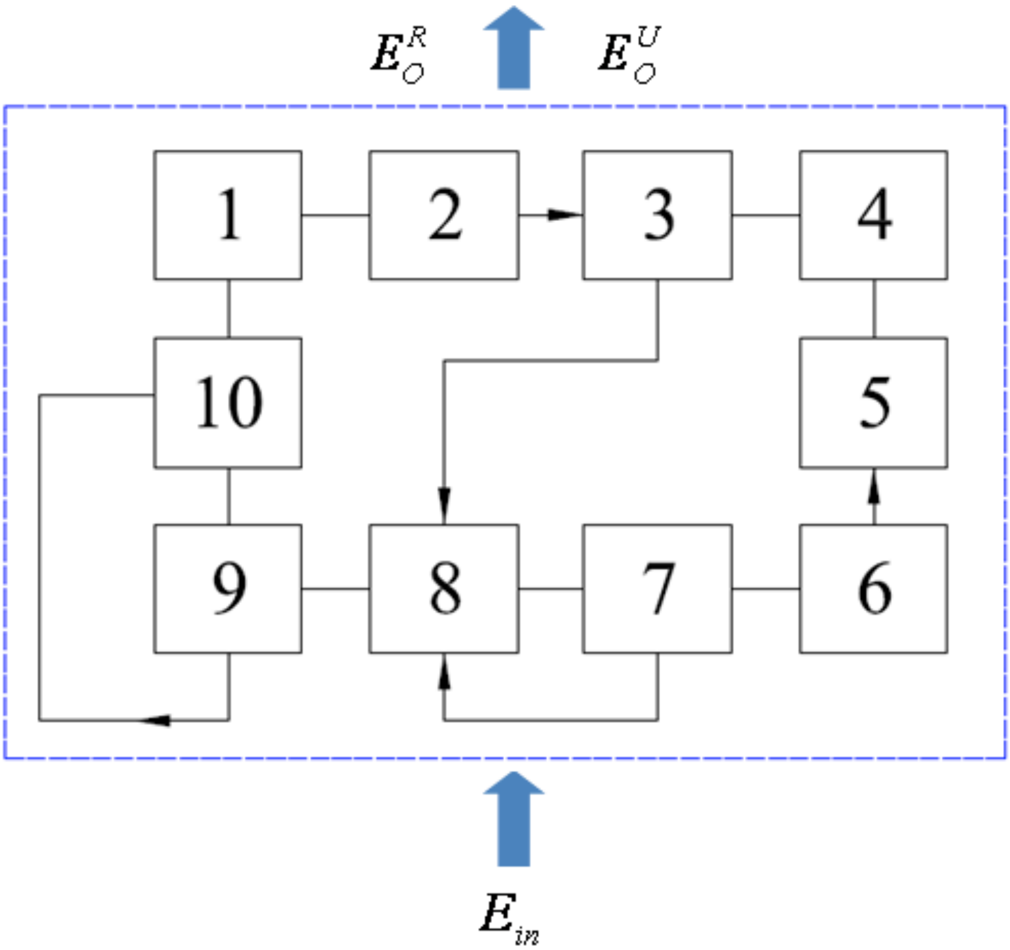


Fig. 2.1: Generic plant diagram

Behaviour of a generic plant can be described by an equation set:

$$\boldsymbol{F}(z) = 0 \tag{2.1}$$

and by an inequalities set:

$$\boldsymbol{D}(z) \geq 0 \tag{2.2}$$

z being the overall variable set:

$$\boldsymbol{z} = \boldsymbol{y} \cup \boldsymbol{b} \cup \boldsymbol{u} \cup \boldsymbol{g} \tag{2.3}$$

$\boldsymbol{y}$ being the variable set:

$$\boldsymbol{y} = \xi \cup \boldsymbol{x} \tag{2.4}$$

made by independent variables ξ *(DOFs)* and by unknown variables $\boldsymbol{x}$.
$\boldsymbol{b}$ being the boundary conditions set (ambient, etc.)
$\boldsymbol{u}$ being the status of the system set made of $\boldsymbol{r}_f$ and $\boldsymbol{a}_f$

$$\boldsymbol{u} = \boldsymbol{r}_f \cup \boldsymbol{a}_f \tag{2.5}$$

g being architecture and geometric data.

Moreover, $\boldsymbol{F}\in\boldsymbol{R}^n$, $\boldsymbol{D}\in\boldsymbol{R}^d$, $\boldsymbol{\xi}\in\boldsymbol{R}^q$, $\boldsymbol{x}\in\boldsymbol{R}^n$, $\boldsymbol{u}\in\boldsymbol{R}^s$, $\boldsymbol{b}\in\boldsymbol{R}^b$, $\boldsymbol{g}\in\boldsymbol{R}^g$

In general equations $\boldsymbol{F}$ are highly non linear and express conservation of mass, momentum, energy and entropy, and other phenomena such as work and heat transfer, combustion, pressure loss, etc. $\boldsymbol{F}$ includes also fluid properties, auxiliary equations, machine and equipment specifications. Equations can also be expressed in terms of graphs or tables. $\boldsymbol{D}$ represents a set of physical, thermal, chemical and geometrical conditions, as well as other constrains which determine the domain where the problem (2.1) solution exists.

The values associate to the vector ξ components usually is establish ξ_k according to suitable criteria one of which can be the search of an appropriate objective function optimum value. x is the solution of (1.1) and (1.2). Of course quantities can be exchanged between $\boldsymbol{x}$ and $\boldsymbol{\xi}$.

r_f and a_f are the vectors of realty functions and actuality ones respectively. *Reality functions* r_{fs} are introduced to accommodate the model to reproduce the existing component behaviour in a reference situation (*New & Clean*). Since during operations the component features behaviour change continuously due to various phenomena leading to performance modification, the model of each component has to be tailored to the new situation. Therefore the models of the major components include suitable *actuality functions* a_{fs} that can represent the actual status of the component. a_f accounts for the deviation of the actual component performance from a condition assumed as the reference.

Thermo-mechanical systems are described adopting a modular approach. This means that each component, or group of components, is described by a module whose structure may be defined by one or more subroutines. Each subroutine contains the model of the corresponding elementary unit (i.e. a compressor blade row, heat exchanger zone and so on). Complex components are built up linking various subroutines as macro-components (i.e. a compressor is built up calling the stator and rotor blade rows, VIGV (Variable Inlet Guide Vane) and OGV (Outlet Guide Vane) routines if needed; waste heat recovery system is built up calling the subroutines for all the requested sections).

2.1 Component and lumped features and performance

RO3 modelling approach is based on a FV lumped feature and performance discretisation of components. The approach is addressed to model any kind of machines and apparatuses made of elementary components such as: compressor rows, expander rows, combustion chambers, heat exchangers, pumps, etc. Taking the compressor as an example, a generic row by row scheme as reported in figure 2.2 is taken into consideration.

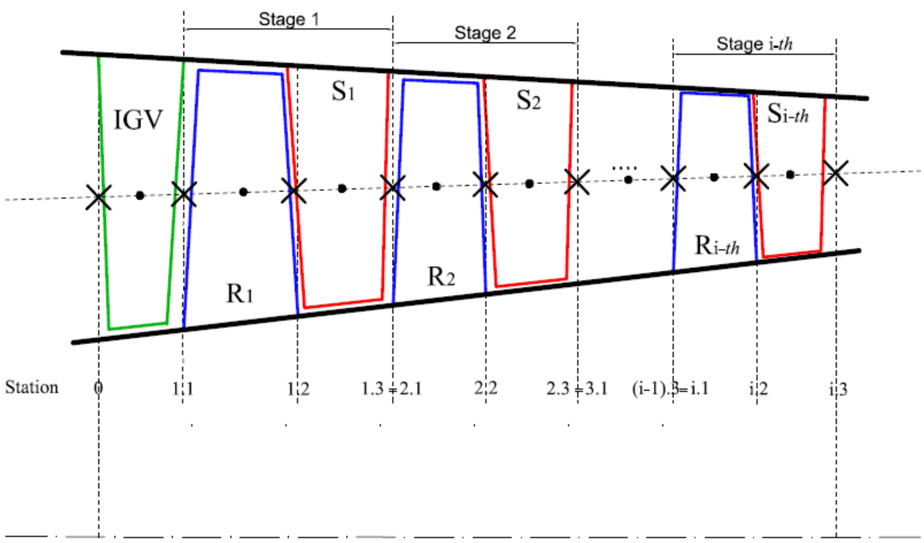


Fig.2.2: Generic compressor row by row scheme

The compressor is divided into each representing a stator or rotor cascade. Each row is included in finite volumes FV's delimited by a boundary, as figure 2.3 shows.

The inlet station and the central node are described by the same number J

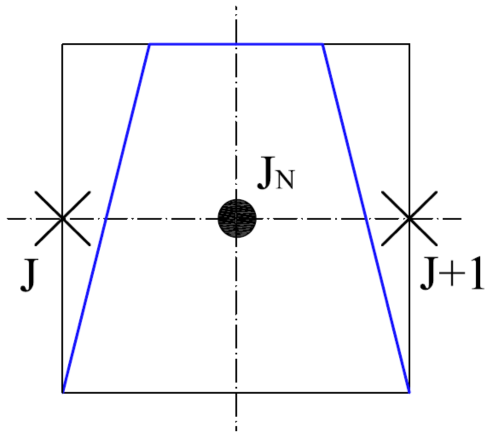


Fig.2.3: Finite Volume Row – Stations and central Node

Real three dimensional time dependent measured flow features are taken into account by lumping on the *FV* boundary models J and $J+1$ the distributions of quantities of interest such as pressure, velocity, temperature, etc., by means of an averaging procedure on surface and time. Moreover the lumping procedure is adapted for the quantities that are involved in the component performance calculation according to the implemented modules. The lumped features are reduced to the *FV* central nodes J_N. A scheme of a compressor model is given in figure 2.2 with the Blade Row Finite Volume scheme given in figure 2.3. Similar approaches are adopted for other devices. e.g. a heat transfer device such as a *shell & tube* heat exchanger can be modelled according to the *FV* elementary device given in figure 2.4a. For a condenser the *FV* approach leads to a multi-zone heat transfer device depicted in figure 2.4b

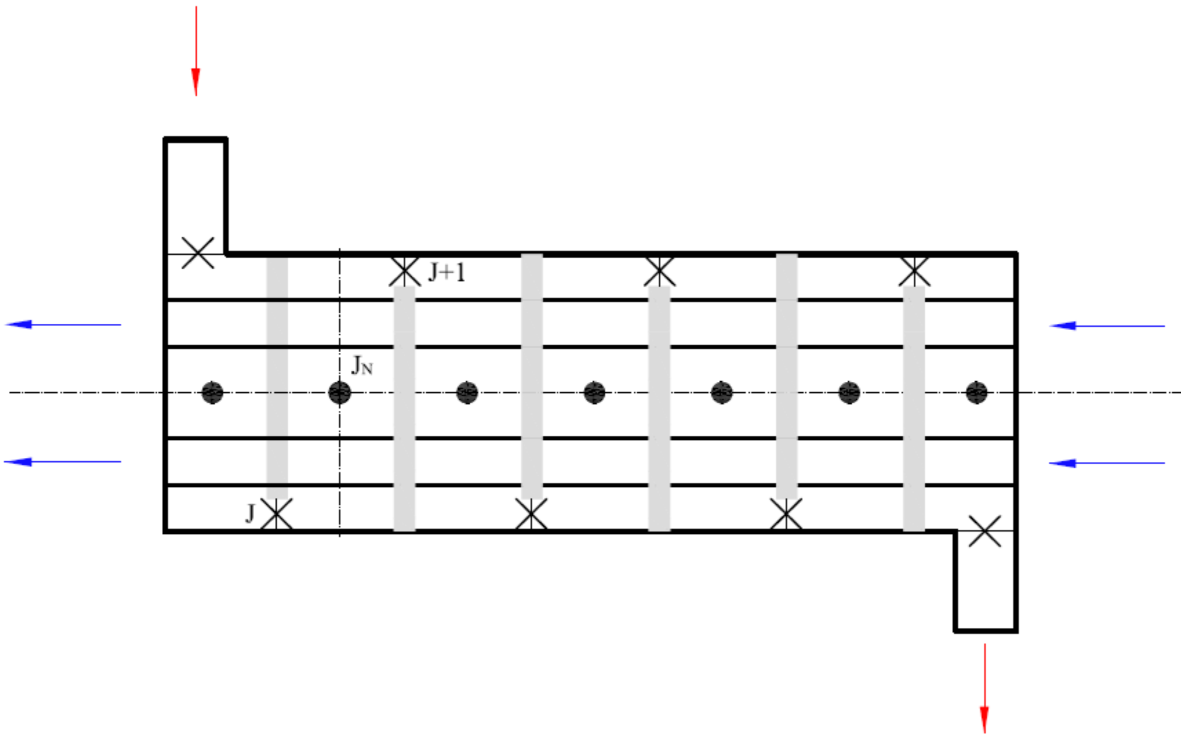


Fig.2.4a: Tube Bundle – Stations and central Node

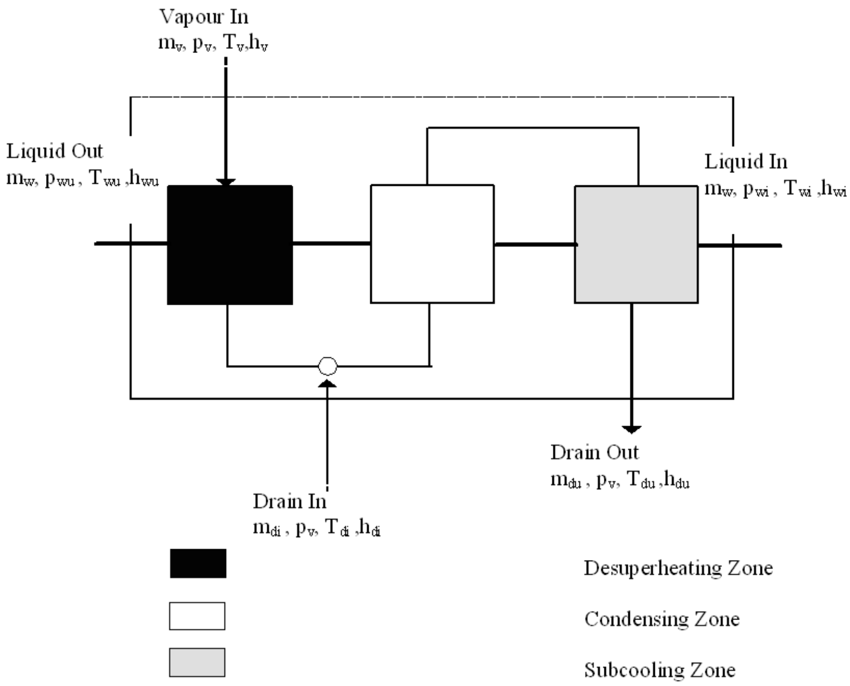


Fig.2.4b: Condenser – Multi-zone heat transfer device

2.2 Methodological Approach

Equations and inequalities (2.1, 2.2, …) describing machines and plant behaviour are addressed to solve different kind of problems. RO3 methodology is based on different steps:

- Cycle Calculation: this procedure is related to preliminary cycle calculation whet the cycle potentials are going to be investigated with only few constraints concerning thermodynamic quantities. Data are usually related to the state of the art machinery and equipment's (i.e. efficiency, heat transfer effectiveness and so on). If related to such above quantities cost specifications are available an optimization procedure can take place. Thermodynamic optimisation is always possible. Indeed overall plant efficiency and specific of work or a combination of these quantities may be chosen as objective function. Results of this calculation are thermodynamic quantities at some plant stations, mass flows, value of powers crossing component boundaries and overall performances.

- Sizing: this phase is preliminary to the next component off design component and plant part load analyses. It consists in the calculation of size of machines and equipment's and alternative global parameters to describe off-design behaviour of components. Input data are from the previous cycle calculations or may come from data base DB related to the commercially available machines and equipment's whose design features are close to that of required cycle calculation. In this phase specifications concerning costs of machines and equipments are used for optimized design. Results of this inverse calculation phase may be devoted to equipment and machine preliminary designs, but at present, they are mainly addressed to the next plant off-design investigation.

- Off-Design Analysis: this direct phase investigation requires the knowledge of geometric data, architecture and some global parameters related to the plant components. Maps of the machine and equipment are obtained and how they match in the plant is studied. Changes in the independent quantities (DOF's) may be investigated according to control policies the related rules may be implemented as specifications. In this casa the component state quantities (u) may be used to optimize operations according with load requirements (electric and thermal power) which are implemented as time dependent constraints.

2.3 Solution strategy

Once the parameters **u** and the degree of freedom **y** have been given, the search of the unknowns **z** may be performed by minimization of the ***plant unbalance function***.

$$\Delta(z) = F(z)^T F(z) \tag{2.6}$$

When the solution of $F(z)$ is achieved (i.e. $z = \bar{z}$), $\Delta(z)$ is zero.
The necessary condition $\Delta(z)$ minimum is achieved, the following **n** equations have to be satisfied.

$$\sum_{j=1}^{n} f_j \frac{\partial f_j}{\partial z_i} = 0 \quad \forall\, i \in [1, n] \tag{2.7}$$

Of course this occurs when:

$$f_j(z^*) = 0 \;\forall\, j \in [1, n] \tag{2.8}$$

In this case, the Hessian matrix of $\Delta(z)$ is definite not negative for $z^* = \bar{z}$

$$\frac{\partial f_j(z^*)}{\partial z_i} = 0 \;\forall\, j \in [1, n] \tag{2.9}$$

In this case z^* is a stationary point for function $f_j(z)$, therefore, it may not be the searched solution point.

The above suggests the idea that stating the following minimization problem

$$\textbf{minimize}\ \{\Delta(z) |\ F(u, z, y) = 0; D(u, z, y) \geq 0; u = u^{\dagger *} \quad y = y^{\dagger *} \} \tag{2.10}$$

The solution of $F(u, z, y) = 0$ with $D(u, z, y) > 0$ is assumed because constraints of the minimization problem are both equations $F(z)$ and inequalities $D(z)$.
At any k-th step, $\Delta(z)$ represents the plant unbalance that vanishes when the solution is achieved.

2.4 Objective function definition

In order to solve problems of sizing, optimization, matching an appropriate algorithm (ECRQP) for the search of the minimum of an objective function has been adopted. In relation to the issues addressed, the objective function takes on different expressions. Indeed a set of objective functions $fob \in R^n$ may be established. The global objective function Fob is

$$Fob = \sum_{j=1}^{N} w_j \cdot fob_j \tag{2.11}$$

The first element, for $j = 1$, represent the unbalance Δ ($\Delta = 1 \cdot fob_1$), the other elements may express a special objectives (like initial cost, operating cost, volume, weight, etc.) and weight vector elements w^T can take the value zero or one. Of course w_1 always must be 1.
Adopting the suitable formulation of the objective function Fob and the vector of unknown quantities z the following problem may be solved:

$$\text{Search } z : \min \left\{ Fob \,\middle|\, F(\xi, x, b, u, g, r_f, a_f) = 0;\ D(\xi, x, b, u, g, r_f, a_f) \geq 0 \right\} \tag{2.12}$$

Matching constraints and therefore plant unbalance are still taking into consideration.

2.5 Solution Methods

Various optimisation techniques based on Equality Constraint Recursive Quadratic Programming (ECRQP), Genetic Algorithms (GA) and Simulated Annealing (SA) as well as hybrid GA-ECRQP and SA-ECRQP have been applied and compared (Cerri, 1996; Cerri et al., 2005; Boccaletti et al., 2000). The choice of the most suitable one depends on the peculiar problem to be solved.

2.5.1 Simultaneous

Simultaneous means that all the unknown variables are foreseen (i.e. each assume a proper value) at the beginning of any step (iteration). Since all the unknown quantities are assumed in the iteration (see fig. 2.5) the contributions of all the component to the objective function (components unbalance, costs, etc.) and to the constraint structure may be calculated. Therefore the plant performance (when it is under an unbalanced condition), costs, emissions of pollutants and the objective function are evaluated. All components are described by equations which express: conservation of energy, mass, momentum and entropy (second thermodynamic law); other phenomena on physical or empirical basis such as work and heat transfer, pressure loss, etc.; fluid properties and auxiliary equations.
Components are described by algebraic relationships and by differential equations which are reduced to algebraic ones by adopting a finite difference procedure. Performance of a plant component is related to its load level. This relationship is influenced by its history (ageing, deterioration, fouling, maintenance and so on).

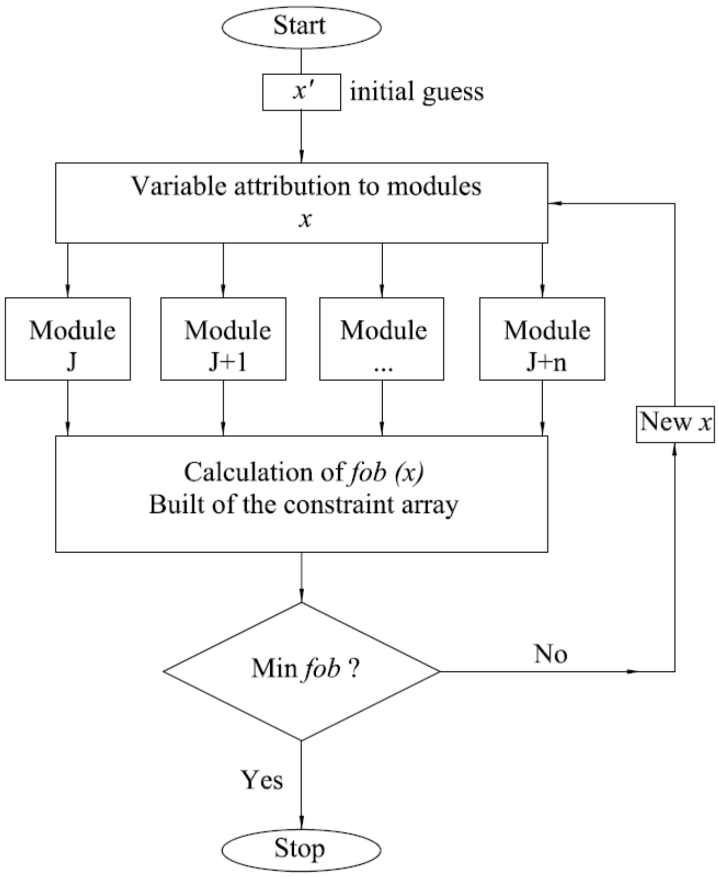


Fig. 2.5 : modular structure calculation method – ECRQP

Problem (1.4) could be solved adopting an optimisation technique developed by Cerri (1991, 2010) based on ECRQP that provides to introduce two merit functions:

- the penalty function:

$$P(z,r) = Fob(z) + \frac{1}{r} \cdot v^T v \qquad (2.13)$$

r being the penalty parameter and v being the vector of active constraints.

- the Lagrange function:

$$L(z,\lambda) = Fob(z) + \lambda^T v \qquad (2.14)$$

λ being the set of Lagrange multipliers related to the constraints.

The parameter r must be positive and when it tends to zero the minimum of $P(z,r)$ tends to the minimum of Fob. The minimum of $L(z,\lambda)$ also coincides with the minimum of Fob.
The solution is found starting from an initial tentative solution x_0. At the generic k^{th} iteration the step d_k (which moves the tentative solution from z_k to $z_{k+1} = z_k + d_k$) is searched by solving a quadratic-programming problem. The objective function is a quadratic approximation of Fob:

$$F_q = f_k d_k + \frac{1}{2} d_k^T d_k H_k \qquad (2.15)$$

f_k being the gradient of the Fob and H_k its Hessian matrix, both evaluated at point z_k. Second order Taylor's series expansion around z_k lead to approximate expression of the penalty function gradient:

$$\nabla P(z_k, r_k) = f_k + H_k d_k + \frac{2}{r_k}(A_k^T v_k + A_k A_k^T d_k) \qquad (2.16)$$

and the Lagrange function gradient:

$$\nabla L(z_k, \lambda_k) = f_k + H_k d_k + A_k^T \lambda_k \qquad (2.17)$$

A_k being the Jacobian matrix of active constraints calculated for $z = z_k$.

The search of d_k is performed by imposing the condition of minimum $P\,(\nabla P(z_k, r_k) = 0)$ and using further conditions resulting by equating the right terms of Eqs. (2.16) and (2.17). Therefore the steps towards the minimum of $Fob\,(z)$ are performed along the locus of penalty function minima, as shown in figure 2.6.

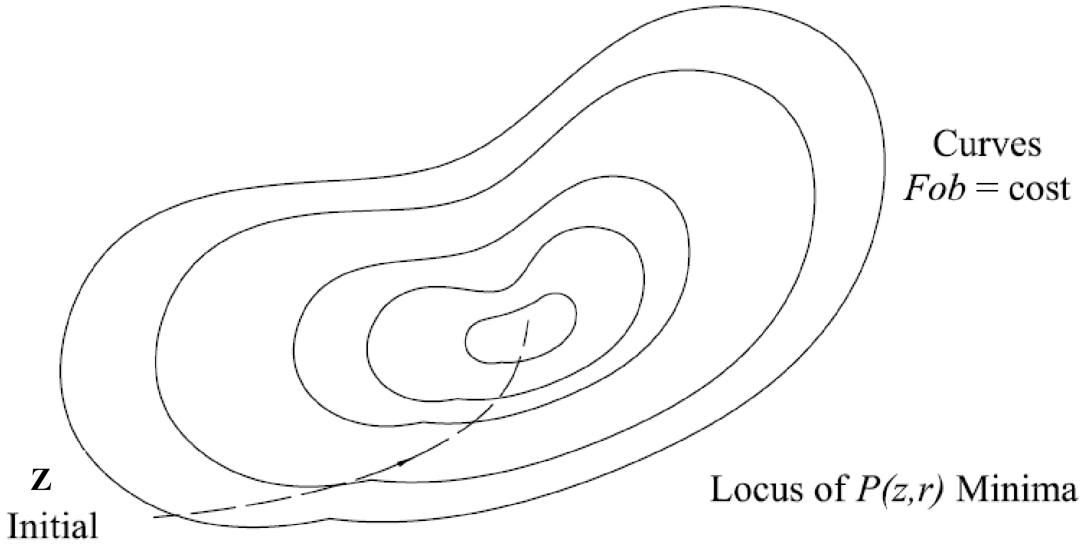


Fig. 2.6: Solution Path along the Locus of *P(z,r)* Minima

2.5.2 Sequential

The most widely adopted method is the sequential one, by this method the plant is divided into modules corresponding to the plant components. For each module subsets of equations and inequalities are established. Each module is analysed sequentially, module outputs are solved from input quantities. Two major aspects related to the computing time have to be pointed out. The first is connected with the non linearity of the module equations which require internal iterations to get outputs. The second is related to closed loops and recycling streams (i.e. when the module under analysis needs other not yet analysed module outputs means that those variables have to be given as tentative ones, therefore external iteration levels in order to have balanced solutions of subsystem process groups). From given data, usually the solution starts from one module and continues following one fluid streams.
Due to the component equations being non-linear and really numerous for complex plants, various level (nested) iterative loops are needed. This method requires a big computation effort and a long CPU time.

2.5.3 Hybrid

The hybrid process consists in the division of the variables into different sets: one is the Dependent Variables *DV* that are the unknowns of the independent equation set; the second variable set consists in the Independent Variables *IV* that have to be given *a priori* and do not change during the calculations. The *IV* set is made of the degree of freedom *DOF's* and of the Boundary Variables *BV or* $\boldsymbol{\beta}$ such as ambient conditions and similar ones. The hybrid approach consists in dividing the calculation environment into two zone. In the first zone the *IV* set is established and the final outputs are saved. The second zone consists in the calculation of the *DV* set using the Non Linear Equation Solution *NLES* that can be performed by a simultaneous or sequential approach. This hybrid methodology is suitable also for the solution of optimization problems. In this case the *DOF* set is divided in two sets. One is $\boldsymbol{\xi}$ that consists in the *DOF*'s to be optimized and the remaining *IV*'s consists in the $\boldsymbol{\beta}$ set whose components β_k remain constant during the calculations.
Accordingly there are three zones:

- the first zone inputs inside the calculation process a suitable ξ^j and calculates the related objective function.
- the second zone inputs into the calculation procedure the $\boldsymbol{\beta}$ set.
- the third zone provides the calculation of the unknowns by a simultaneous or a sequential procedure

Maps of the plant can be calculated by suitably changing the point inside the $\boldsymbol{\beta}$ domain.

The above procedure is implemented by RO3 Research Unit by adopting Genetic Algorithm *GA*, Simulated Annealing *SA* and *ECRQP*. The *GA-ECRQP* hybrid algorithm is schematically represented in figure 2.7.

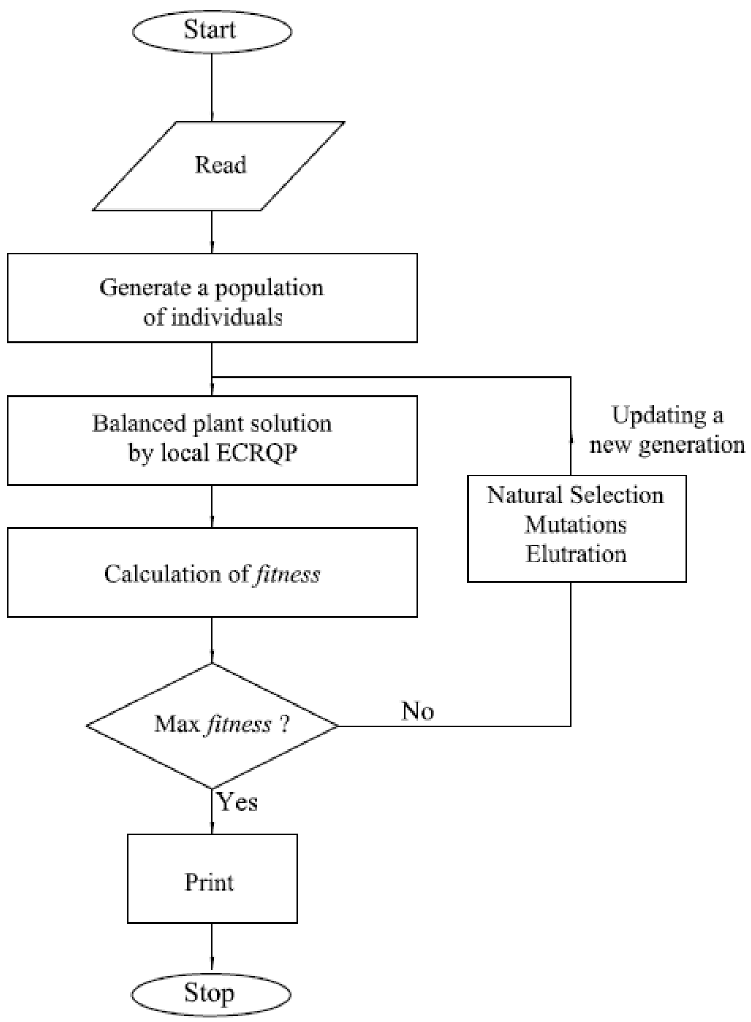


Fig. 2.7: Hybrid methodology – Genetic Algorithm/ECRQP

2.6 Modular approach

With reference to section 2.1, a thermo-mechanical system may be described by modules each corresponding to a component or sub-component of a plant. This is achieved by splitting the **F** (equations) and **D** (inequalities) function vectors in subsets of equations

$$\Phi_j(d^{MJ}, x^{MJ}) = 0 \tag{2.18}$$

and a subset of inequalities

$$\delta_j\,(d^{MJ}, x^{MJ}) = 0 \tag{2.19}$$

Of course the above subsets must satisfy the following conditions

$$\boldsymbol{F} = \Phi_1 \cup \Phi_2 \cup \Phi_j \cup \Phi_z \tag{2.20}$$

and

$$\boldsymbol{D} = \delta_1 \cup \delta_2 \cup \delta_j \cup \delta_z \tag{2.21}$$

Each pair $\boldsymbol{\Phi}_j$, δ_j represents a module that can be a real component or a fictitious one; z being the number of modules.

A module is schematically represented in fig.1.3 variables x^{MJ} are input; output quantities are:

values assumed by equalities $\boldsymbol{\Phi}_j$, and inequalities δ_j;

the partial plant unbalance (component unbalance) $\boldsymbol{\Delta}_j$;

other partial objective functions which represent component contributions fob_j

other quantities of interest.

Due to the peculiarities of the simultaneous solution method, implementation of component specifications which generally are function of both component inlet and outlet quantities is really simple in these modules.

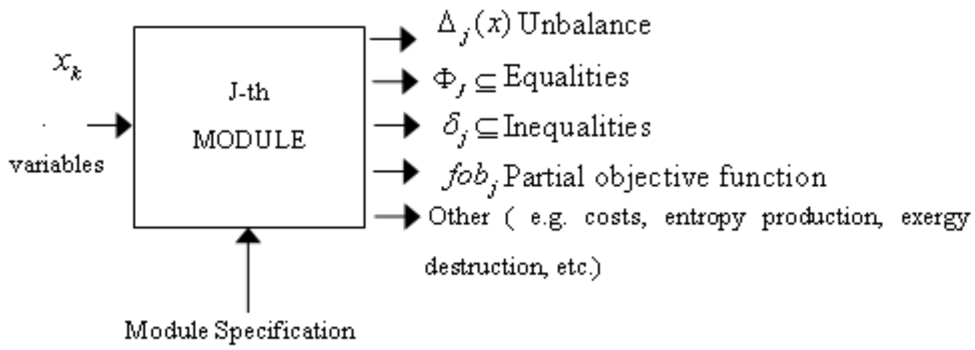


Fig. 2.8: Schematic Module Specification

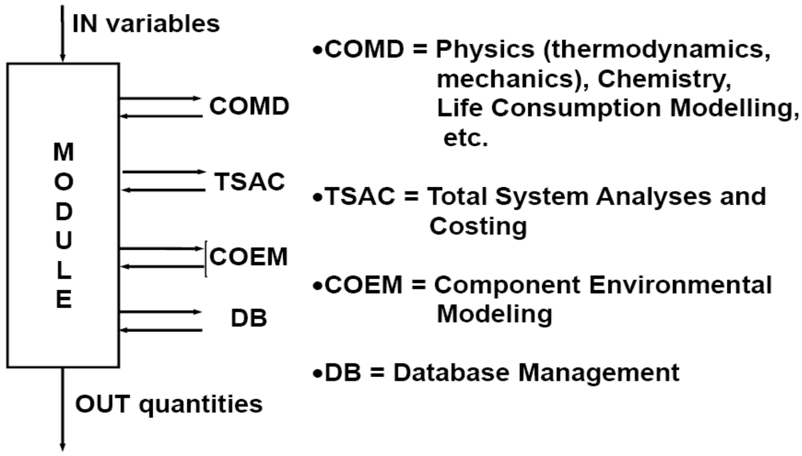


Fig. 2.9: block showing the connections between the module and the sections which produce the single objective function contribution

Fig. 2.9 shows the connections between the sections which treat thermodynamic, mechanical, emission, cost and control aspects. Each module may activate the sections that are of interest for the present study, each section produces the single objective function contribution.

The single module calls the COMD sections (TH and ME in fig 2.9). The results of COMD are given to the other sections. TSAC and COEM contribute to the module objective function. A section of program built the global objective function by the single module contributions and calls the solution routine.

3 List of models

The list below shows the models that have been developed by RO3 on his own. These models can be adapted to the H2-IGCC plant.

- Weather Hood
- Air Filter
- Evaporative Cooler
- Coil
- Compressor
- Combustion Chamber
- Expander
- Diffuser
- Ducts connecting GT components
- Steam generator 1 (fired)
- Steam generator 2 (fired)
- Heat Recovery Steam Generator (HRSG)
- Superheater
- Boiler
- Economizer/Preheater
- Postfire
- Steam Turbine (1)
- Steam Turbine (2)
- Steam/water surface heat exchangers
- Deaerator
- Storage Vessel
- Pump
- Attemperator
- Gassifier
- Process compressors, fans and pumps
- Fluidized beds
- Heat Exchangers
- Junctions
- Valves
- Splitters
- Mixers
- Electric power generator
- fluids properties

4. Description of the models

4.1 Fluid Properties

Fluid properties routines have been derived by those already existing at RO3. Improvements concerning easy and fast utilization and more accurate description (such as the influence of ambient pressure on humid air properties). The amount of water vapour existing in the air varies with different conditions. Gas mixtures (e.g. O_2, N_2, CO_2, H_2O) with emphasis to air and combustion gases have been taken into consideration.
In this context revision of fluid properties have been done to identify the state of the flow (which may be in a mono-phase or two-phase flow) and routines have been improved to calculate thermodynamic properties at different conditions. Special modifications have been carried out by RO3 to allow the functions calculated from the two sites, wet and superheated, to have the same value on the dry saturated steam line.

4.1.1 Properties of two phase flow system

As discussed above, the air contains water, which could be in the form of vapour phase and liquid phase. Thermodynamically it is a difficult task to obtain the properties of the mixture undergoing a phase change. However assuming thermodynamic equilibrium for the system the properties of the mixture can be obtained, based on the following concepts:

Specific heat at constant pressure for two phase flow

The specific heat of a two phase flow mixture at equilibrium is defined as the rate of change of enthalpy with respect to temperature at constant pressure, i.e.

$$c_p = \left[\frac{dH}{dT} \right]_p \tag{4.1}$$

Since the enthalpy of the mixture at equilibrium is the sum of the enthalpies of the chemical species in the gas phase and enthalpies of the liquid phase, i.e.

$$H = m_g h_g + m_l h_l \tag{4.2}$$

where (g) and (l) denotes the gas phase and liquid phase in the system.

$$h_g = f\,(T_i,\ p_i,\ \text{mass fraction of chemical species in the gas phase}) \tag{4.3}$$

$$h_l = f\,(T_i) \tag{4.4}$$

Thus the specific heat of the system is given by

$$c_{p_{sys}} = \frac{d(m_g h_g)}{dT} + \frac{d(m_l h_l)}{dT} \tag{4.5}$$

being

$$m_g + m_l = \text{constant}$$

which implies that

$$\frac{dm_g}{dT} = -\frac{dm_l}{dT}$$

thus, eq. 4.5 can be written as

$$c_{p_{sys}} = m_g \frac{dh_g}{dT} + m_l \frac{dh_l}{dT} + (h_l - h_g)\frac{dm_l}{dT} \tag{4.6}$$

Note that the first term in the right hand side of eq. 6 indicates the specific heat of the chemical species in the gas phase, the second term indicates the specific heat of liquid phase, and the last term indicates the rate of change of liquid to vapour phase at constant pressure.
Hence the specific heat of the system can be expressed as:

$$c_{p_{sys}} = \frac{H_2 - H_1}{T_2 - T_1} \tag{4.7}$$

where (H_2) denotes enthalpy at temperature (T_2)and (H_1) is the enthalpy at (T_1).

Specific heat at constant volume for two phase flow

The specific heat of a two phase flow mixture at equilibrium is defined as the rate of change of internal energy with respect to temperature at constant volume, i.e.

$$c_V = \left[\frac{dU}{dT} \right]_V \tag{4.8}$$

Since the internal energy of the mixture at equilibrium is the sum of the internal energy of the chemical species in the gas phase and internal energy of the liquid phase water, i.e.

$$U = m_g u_g + m_l u_l \tag{4.9}$$

Being:

$u_g = h_g - R_g T_i$ (for gas phase flow)

$u_l = h_l - \frac{p_{sat}}{\rho_W}$ (for liquid phase flow)

R_g = f(T_i, p_i, mass fraction of chemical specious in the gas phase)

ρ_W = f(T_i) p_{sat} = f(T_i)

(P_{sat}) is the saturation pressure and (ρ_W) is density at liquid water saturation.

The specific heat of the system is given by

$$c_{Vsys} = \frac{d(m_g u_g)}{dT} + \frac{d(m_l u_l)}{dT} \tag{4.10}$$

which can be expand as

$$c_{Vsys} = m_g \frac{du_g}{dT} + m_l \frac{du_l}{dT} + (u_l - u_g) \frac{dm_l}{dT} \tag{4.11}$$

Again the first term in the right hand side of eq. 4.11 indicates the specific heat of the chemical species in the gas phase, the second term indicates the specific heat of liquid phase, and the last term indicates the rate of change of liquid to vapour phase, at constant volume.

$$c_{Vsys} = \frac{U_2 - U_1}{T_2 - T_1} \tag{4.12}$$

where (U_2) is the internal energy at temperature (T_2) and (U_1) is the internal energy at temperature (T_1).

Density of a two phase flow system

The density of a system is defined as the mass per volume of the mixture and is given as:

$$\rho = \frac{m}{V} \tag{4.13}$$

The volume of the chemical species in the gas phase of the system is function of pressure, temperature and gas constant R_G, and gas phase composition.

V_g = f(p_i, T_i, R_G, gas phase composition) (4.14)

Also the volume of liquid phase is function of temperature and saturation pressure, i.e.

V_l = f(T_i) (4.15)

Thus the total volume of system is

$$V_{sys} = V_g + V_l \tag{4.16}$$

The density of the system per kg of mass is

$$\rho_{sys} = \frac{1}{V_{sys}} \tag{4.17}$$

<u>sound velocity for a two phase flow system</u>

All the substances are compressible. The compressibility of any fluid id defined by the Bulk Modules of Elasticity denoted by (Ks) and is given by

$$Ks = \rho\left(\frac{dP}{d\rho}\right) \tag{4.18}$$

The sound velocity for the chemical specious of the gas phase in the system is

$$C = \sqrt{\gamma R_g Ti} = \sqrt{\frac{K_g}{\rho_g}} \tag{4.19}$$

where (g) denotes the chemical specious of gas phase in the system. Thus the bulk's modules of elasticity for the gas phase is given by

$$K_g = \rho_g \gamma R_g Ti \tag{4.20}$$

Hence the sound velocity of the system containing liquid water can be expressed as (Alan Vardy, 1990);

$$C_{sys} = \sqrt{\frac{\rho_g \gamma R_g Ti}{\rho_{sys}}} \tag{4.21}$$

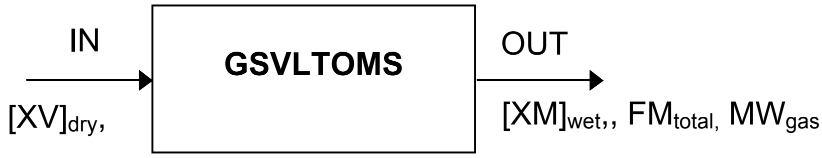
The speed of sound in a mixture containing liquid water is much smaller than the speed of gas phase alone. In a mixture containing liquid water the compressibility factor is governed by the air whereas the density is governed by the water.

4.1.2 Fluid Property routines developed

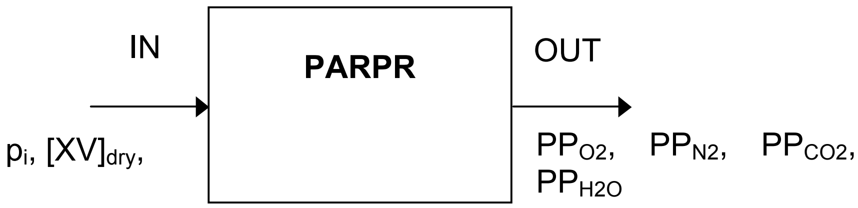
To show the fluid properties behaviour with different chemical composition and at various physical conditions , which may be encountered in practice, the following routines have been developed to meet the necessity of having an inlet temperature ranging from -50°C to 60°C for various humidity conditions.

- ***<u>VOLTMAFR</u>* <u>(volume to mass fraction routine)</u>** for a given ambient condition (i.e. dry volumetric compositions $[XV]_{dry}$, (consisting of O_2, N_2, CO_2, H_2O), relative humidity [φ], pressure [p_i], temperature [T_i]). This routine evaluates the mass fractions of the mixture $[XM]_{wet}$ (consisting of O_2, N_2, CO_2, and H_2O).

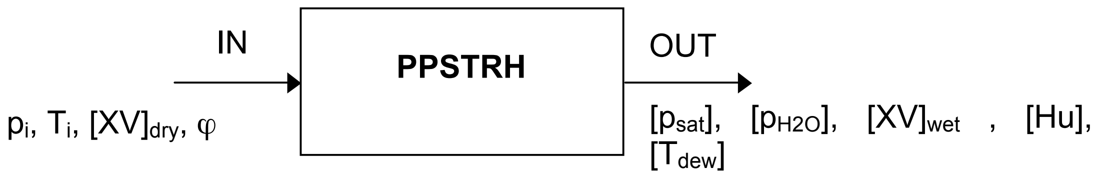
- ***GSVLTOMS* (gas volume to mass fraction)** this routine evaluates the mass fractions of the mixture $[XM]_{wet}$ (consisting of O_2, N_2, CO_2, H_2O), total mass fractions(FM_{total}) and gas molecular weight (MW_{gas}), for a given volumetric fractions of the mixture $[XV]_{dry}$, (consisting of O_2, N_2, CO_2, and H_2O).



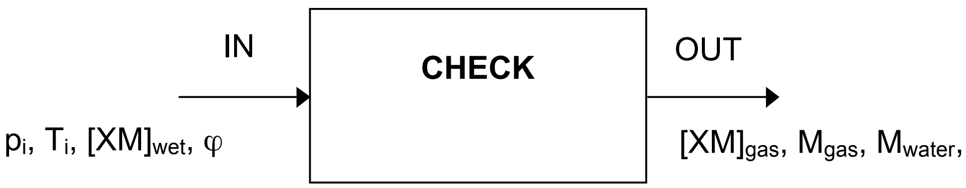
- ***PARPR* (partial pressure of the gas mixture)** this routine evaluates the partial pressure of different components in the mixture (i.e. PP_{O2}, PP_{N2}, PP_{CO2}, PP_{H2O},) for a given pressure(p_i), and volumetric fractions of the mixture $[XV]_{dry}$, (consisting of O_2, N_2, CO_2, H_2O).



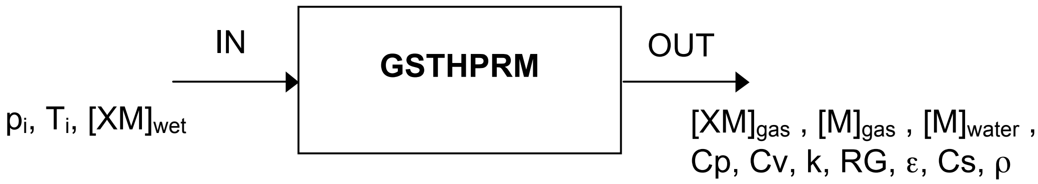
- ***PPSTRH*** given ambient condition (i.e. dry volumetric compositions $[XV]_{dry}$, (consisting of O_2, N_2, CO_2, and H_2O), relative humidity[φ], pressure [p_i], and temperature[T_i]). This routine evaluates the saturation pressure[p_{sat}], partial pressure of vapour [p_{H2O}], volumetric compositions of the mixture $[XV]_{wet}$ (consisting of O_2, N_2, CO_2, and H_2O), humidity ratio [Hu], and dew point temperature of the mixture [T_{dew}].



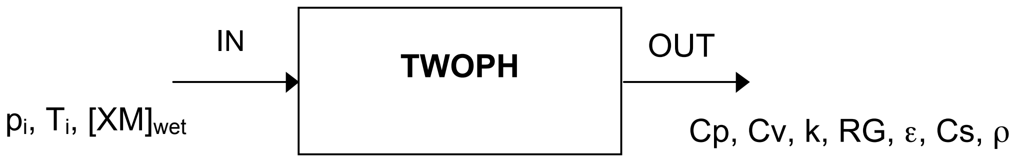
- ***CHECK* (subroutine check)** for a given pressure(p_i), temperature(T_i), and fractions of mass compositions in the mixture $[XM]_{wet}$ (consisting of O_2, N_2, CO_2, and H_2O). This routine checks that the system is a mono-phase or two-phase flow. If there is a possibility of existence of water in the system it calculates the amount of water in the system (M_{water}), total mass of chemical species in the gas phase of the system (M_{gas}), and gas mass fractions of each chemical species in the system (XM_{gas}).



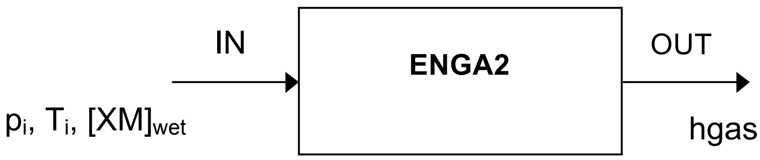
- ***GSTHPRM* (gas thermodynamic properties)** for a given pressure(p_i), temperature(T_i), and mass fractions of gas mixture $[XM]_{wet}$ (consisting of O_2, N_2, CO_2, H_2O). This program is used to recognize the mixture condition (i.e. is it a mono-phase or two-phase flow). If it is a mono-phase flow the routine calculates the mass of each chemical species in the gas phase. If there is a possibilities of existing of liquid water, it calculate the mass of liquid water (M_{water}), and the mass fractions of each chemical species in the gas phase of the system (XM_{gas}). In addition the routine calculate the thermodynamic quantities of the mixture (i.e. specific heat at constant. pressure [Cp], specific heat at constant volume [Cv], gas constant[RG], specific heat ratio of the system [k], polytropic exponent of the mixture [ε], sound velocity [Cs], and density of the gas mixture for the-mono phase flow or two-phase flow [ρ].



- ***TWOPH* (two phase flow)** for a given pressure(p_i), temperature(T_i), and fractions of wet mass compositions in the gas mixture $[XM]_{wet}$ (consisting of O_2, N_2, CO_2, H_2O). This routine obtains the thermodynamic properties of the mixture, when some amount of water exists in the mixture (i.e. specific heat at constant pressure[Cp], specific heat at constant volume [Cv], specific heat ratio of the system [k], polytropic exponent of the wet mixture [ε], density of the mixture [ρ], and sound velocity [Cs]).

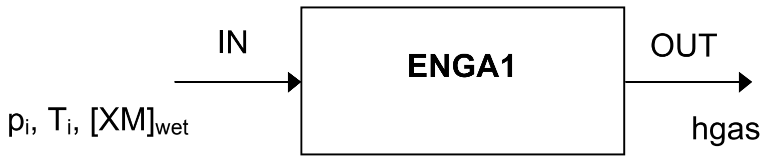


- ***ENGA2* (enthalpy of gas mixture)** this routine calculate the specific enthalpy of gas phase mixture for a given pressure (p_i), temperature (T_i), and fractions of compositions $[XM]_{wet}$ (consisting of O_2, N_2, CO_2, H_2O). In this routine the heat of vaporization of water vapour is not considered.



- ***HGA2* (gas enthalpy)** calculate the specific enthalpy of a gas mixture for a given temperature (T_i), and mass fraction of compositions $[XM]_{wet}$ (consisting of O_2, N_2, CO_2, H_2O). In this routine the heat of vaporization of water vapour is not considered.

- ***ENGA1* (enthalpy of gas mixture)** this routine calculate the specific enthalpy of gas phase mixture for a given pressure (p_i), temperature (T_i), and fractions of compositions $[XM]_{wet}$ (consisting of O_2, N_2, CO_2, H_2O). In this routine the heat of vaporization of water vapour is being considered.



- ***HGA1* (gas enthalpy)** calculate the specific enthalpy of a gas mixture for a given temperature (T_i), and mass fraction of compositions $[XM]_{wet}$ (consisting of O_2, N_2, CO_2, H_2O). In this routine the heat of vaporization of water vapour is being considered.

- ***GASCO*** for a given air compositions $[XM]_{wet}$ (consisting of O_2, N_2, CO_2, H_2O), fuel constituents $[XM]_{fuel}$ (consisting of C, H_2, O_2, N_2, S, H_2O), and air-fuel ratio (α).
  This routine determines
    1) fractions of gas products at combustion chamber exit for burning 1 kg of fuel $[XM]_{wet}$ (consisting of O_2, N_2, CO_2, H_2O).
    2) stoichiometric air-fuel ratio, and
    3) % of excess air.

- ***SPHTG*** calculate the specific heat of the gas phase at constant pressure for a given gas temperature and gas compositions (consisting of O_2, N_2, CO_2, and H_2O).

Accordingly, computer programmes have been written for each of the above cases, incorporating correlations for calculating the gas properties, by implementing the concepts discussed above. The list of input and output quantities developed for each of the above routine and the other ones available are given in appendix A.
Moreover already existing water steam thermodynamic functions have been improved to have a shorter computing time. Modifications to traditional subroutines have been introduced, to have an exact matching of enthalpy values (and of other quantities) on both saturated water and saturated steam lines when such quantities are calculated with functions valid in two adjoining regions. In order to calculate the thermodynamic quantities below the triple point, properties of solid phase and solid-vapor mixture phase have been added to the routines. The list of input and output quantities developed for each routine is given in appendix B.
The flow diagrams given in appendix C shows the details of calculation procedure for the following cases:
Analysis of fluid properties for a mono phase flow system.
Recognise the system is a mono phase or two phase flow and calculate the mass of gas phase, mass of liquid phase, and mass fraction of each chemical species of the gas phase in the system.
Analysis of fluid properties for a two phase flow system.

4.2 Equivalent thermodynamic quantities

The calculation of gas turbine processes usually are performed iteratively due to the non-linear relationships which relate the working fluid thermodynamic and transport properties to pressure and temperature. When optimizations are performed, thus a significant amount of computing resources may be saved if there is a straight forward calculation may be performed as a function of process parameters.
Moreover in on line systems with close loop controls, if load varies to operate the plant optimally certain input parameters has to be changed accordingly. Under such circumstances it is desirable that the computing time will be as low as possible. Thus as soon as the load varies, computer may calculate the optimum parameters for the new operating conditions to be fed back to control unit to operate the plant optimally. Hence there is a need to find new techniques which can reduce the computing time significantly.
A fast direct calculations of fluid polytropic exponents across compressors and gas expanders, as well as specific heats across combustion chambers fed with different fuels equivalent to those of real processes have been studied. Such quantities are expressed as functions of parameters relevant for gas turbine calculations. In this context various mathematical algorithm and computational methods that were carried out for calculation of gas turbine gas properties have been reviewed and summarized below:

4.2.1 Background Concepts

In general compression and expansion process thermodynamic differential equations may be expressed as:

$$\frac{dT}{T} = \gamma\varepsilon\frac{dp}{p} \tag{4.22}$$

where polytropic exponent $\varepsilon = R/C_p$ is function of pressure(p), temperature(T) and fluid composition(XM):

$$\varepsilon = f_1(p, T, [XM]_{fluid}) \tag{4.23}$$

For expansion $\gamma = \eta_t$ and for compression $\gamma = 1/\eta_c$ take into account the polytropic efficiencies. The fluid compositions at compressor inlet consists of different compounds, which may be in the form of gas or vapour, such as H_2O. Thus it is of great importance to recognize the H_2O status, to evaluate the effect of vapour pressure on the gas constant of the mixture.
Owing to relationship (4.23) for ε, eq. (4.22) may be integrated by a step-by-step process. Starting from the initial point p_1, T_1 to the final point p_2. If the number of steps are infinite the final integral value $t_{2\infty}$ is the true value. In the step-by-step integration the number of steps has been fixed according to the deviate of T_{2r} with the step number to check the asymptotic value. As consequence T_{2r} has been assumed as true value. A high degree of accuracy in the results may be achieved with many steps.
Since in a perfect gas the value of ε is a constant value independent of pressure and temperature, the integration of eq. (4.22) leads to the expression:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\gamma\varepsilon} \tag{4.24}$$

that suggests the concept of equivalence. This means that taking into account the behaviour of real gases and vapour it may be established an equivalence between a real gas whose extremes in the integral time are p_{1r}, T_{1r}, p_{2r}, T_{2r} and a perfect fictitious gas whose value of equivalent polytropic exponent (ε_{eq}) bring to the following equation;

$$\varepsilon_{eq} = \frac{\ln\frac{T_{2r}}{T_{1r}}}{\gamma\ \ln\frac{p_{2r}}{p_{1r}}} \tag{4.25}$$

of course the equivalence will establish a relationship between ε_{eq} and the variable parameters involved in the process, which are:

$$\varepsilon_{eq} = f_2(p_1, T_1, \beta, \gamma, [XM]_{fluid}) \tag{4.26}$$

where p_1 and T_1 are the fluid inlet pressure and temperature, for compression $\beta=p_2/p_1$ and for expansion $\beta=p_1/p_2$. The fluid composition at compressor inlet is consisting of dry air volumetric compositions with a given relative humidity(φ).

The relationships (4.25) and (4.26) may be found by numerical procedures which give the value of ε_{eq} in a domain whose points are characterized by parameters of eq. (4.26) within a proper range of variability.
Similarly the value of equivalent specific heat (C_{eq}), using energy balance across combustion chamber (Cerri, 1982) is defined as:

$$C_{eq} = \frac{\eta_{c.c.}\ \Gamma_i}{(1+\alpha)\ (T_3 - T_2)} \tag{4.27}$$

C_{eq} is the specific heat of a fictitious working fluid whose (α+1) mass units flow through the combustion chamber entering at temperature(T_2) and exit at temperature(T_3) after it has received the heat $\eta_{c.c.}\Gamma_i$.
The value of Ceq can be established depending on the parameters involved in the process, i.e.

$$Ceq = f_3(p_2, T_2, \Delta T_{c.c.}, \eta_{c.c}, [XV]_{dry\ air}, \varphi, [XM]_{fuel}, \Delta p_{c.c.}/p_{in}) \tag{4.28}$$

where $\Delta T_{c.c.}$ is the temperature difference between the gas at exit(T_3) and air at inlet(T_2) of combustion chamber. The value of C_{eq} in eq. (4.27) is obtained within the domain characterized by parameters of eq. (4.28) in a proper range of variability.
The value of equivalent polytropic exponents for compressor and gas expander (ε_{eq}), and the equivalent specific heat across combustion chamber (Ceq) are calculated separately for n_p(more than 2000 points) sets of random variables. The random sets of variables have been generated using a Universal Random Number Generator (Forsythe "et al.", 1997) within the physical range of each component.
The unknown quantities are the regression coefficients which are assigned by optimization method to calculate the equivalent quantities directly for the random set of variables given in eq. (4.26) and (4.28) with different polynomial order functions.
The function adopted to find the regression coefficients is polynomial addition with a single constant;

$$F_{eq}(\xi) = c_{11} + \sum_{K=1}^{N_k} \sum_{J=1}^{NO_k+1} a_{KJ}\, \xi_K^{J-1} \tag{4.29}$$

being $$\xi_K \in \left[1 < NVAR < N_K\right]$$

The solution method adopted to find the regression coefficients is ECRQP. This method has been described in detail by Cerri (1996).

4.2.2 Step-by-Step Integration of Compressor and Gas Expander

Using finite difference concept form of polytropic transformation, an equivalent method have been proposed to calculate the exit condition of compressor and gas expander using step-by-step integration. This method is based on a variable pressure difference bringing to roughly constant temperature increment.
Consider a thermodynamic process between two ends statuses(1 and 2). Let the process is divided into a number of small intervals. Using the concept given in eq. (4.22) the exit temperature from each interval is expressed:

$$T_{j+1} - T_j = \frac{T^*(\gamma \quad \varepsilon^*)}{p^*}(p_{j+1} - p_j) \tag{4.30}$$

The value of $(T_{j+1}$-$T_j)$ remains almost constant along the integral, also $(p_{j+1}$-$p_j)$ has been chosen accordingly. When the number of steps are sufficiently high the value of T^*, p^*, and ε^* may be assumed as a arithmetic average values of each step. If n_p points are assumed n_p-1 of eq. (4.30) may be written.
Assuming p_1, T_1, p_{np} and consequently p_j $(n_p$-2) the temperatures T_j $(n_p$-1) $2\leq j \leq n_p$ may be calculated. To have a roughly constant temperatures difference $(T_{j+1}$-$T_j)$, the pressures p_j $2\leq j \leq n_p$-1 are chosen according to,

$$p_{j+1} = p_j + \left[\frac{p_j}{T_j} C_1\right] \tag{4.31}$$

C_1 control the number of intervals, and is being a suitable constant which tends to zero when n_p tends to infinite. Higher the number of integration steps lower is the value of C_1. The above calculation have been performed in a domain of definition of points $P_k(p_1, T_1, \beta, \gamma, [XM]_{fluid})$, where k is sufficiently high. Calculation points have been randomly spread in the domain for any point (k).
Based on the above procedure a value of $T_{2\infty,k}$ has been calculated. Also $\varepsilon_{eq,k}$ is calculated using eq. (4.25) with $T_{2\infty,k}$=$T_{2r,k}$. Using eq. (4.29) and solving $T_{2e,k} = T_{1k}\,\beta^{\gamma\varepsilon_{eq}}$ the error on compression and expansion has been established as:

$$Err = \frac{1}{N_{max}} \sum (T_{2\infty,k} - T_{2e,k})^2 \tag{4.32}$$

Coefficients in eq. (4.29) have been found by mini-max error within a specified domain.

4.2.3 Combustion Chamber Calculation using Equivalent Method

The combustion chamber is treated as a system producing a pressure drop with energy, mass and momentum conservation and with chemical reaction.
Chemical reactions are assumed to be at equilibrium. Stoichiometric equations have been implemented to account for oxidation of components.
Air enter into combustion chamber(c.c.) with the following specifications (see fig. 4.1):
fraction of dry volumetric compositions $[XV]_{dry}$ consists of O_2, N_2, CO_2, and H_2O, with a given relative humidity (φ),
air inlet pressure (p_2) and temperature (T_2),
Given fuel specifications,
mass fractions of fuel $[XM]_{fuel}$ consisting C, S, H_2, O_2, N_2, and H_2O(vapour),
fuel lower heating value (Γ_i),
fuel enthalpy (h_f), and
air-fuel ratio (α).
Also for a given combustion chamber design,
combustion chamber efficiency ($\eta_{c.c.}$), and
pressure drop across combustion chamber ($\Delta p_{c.c.}/p_2$).

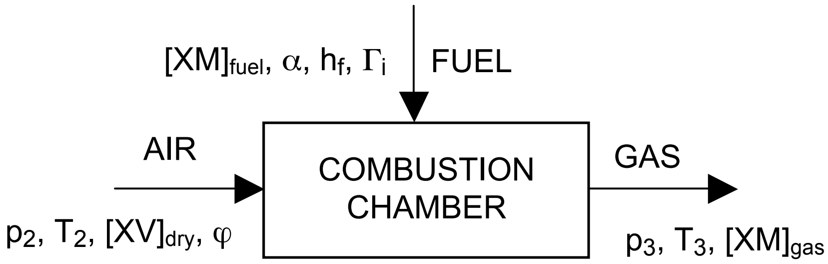


Fig. 4.1Combustion Chamber Specifications

The exit quantities from combustion chamber can be determined as follow,
combustion chamber exit pressure is a function of,

$$p_3 = f_4(p_2, \Delta p_{c.c.}/p_2) \tag{4.33}$$

enthalpy of air at combustion chamber inlet,

$$h_2 = f_5(p_2, T_2, [XV]_{dry,} \varphi) \tag{4.34}$$

fractions of mass compositions of the product gases $[XM]_{gas}$ at c.c. exit,

$$[XM]_{gas} = f_{12}([XV]_{dry,} \varphi , [XM]_{fuel} , \alpha) \tag{4.35}$$

The enthalpy of product gases (h_3) is obtained from heat balance across combustion chamber, using the mass and enthalpy concept, i.e.

$$h_3 = \frac{\alpha h_2 + h_f + \eta_{c.c.}\Gamma i}{(1+\alpha)} \tag{4.36}$$

The product gases temperature (T_3) is achieved iteratively using equality constraint that has to satisfy the state point enthalpy of gases.
The minimum error across combustion chamber is achieved from the difference;

$$Err = \frac{1}{N_{max}} \sum (\Delta T^*_{c.c.} - \Delta T_{c.c.})^2 \tag{4.37}$$

Solving eq. (4.27) $\Delta T_{c.c.}$ is calculated, also $\Delta T^*_{c.c.}$ is obtained from regression coefficients of eq. (4.29). Coefficients leading the mini-max error in the domain.
The list of programs developed for the above procedures are listed in appendix D.

4.3 Component models

4.3.1 Weather Hood

Figure 4.2 shows the weather hood scheme:

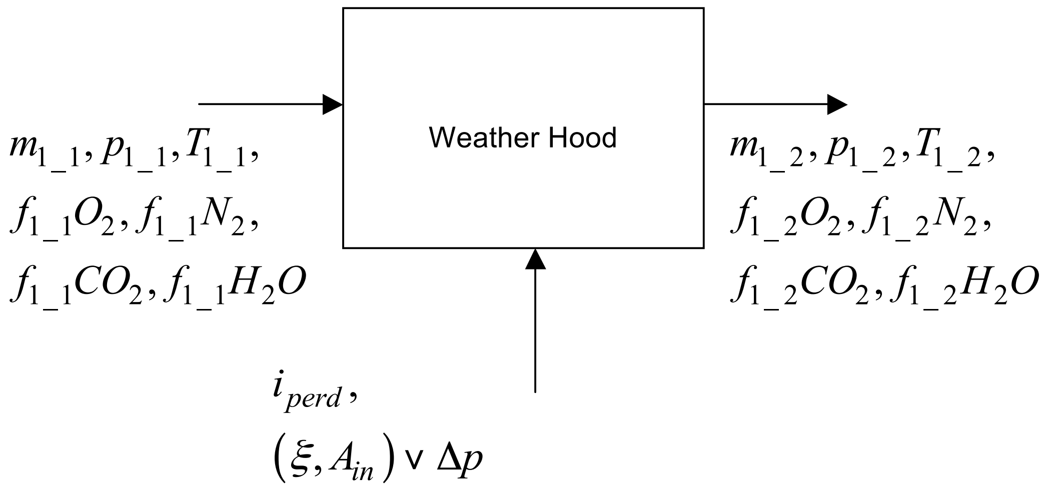


Fig. 4.2: weather hood scheme

In the following table module inputs and outputs are described:

Inlet air:		Outlet air:	
• mass flow	• m_1.1	• mass flow	m_1.2-a
• pressure	• p_1.1	• pressure	p_1.2-a
• temperature	• T_1.1	• temperature	T_1.2-a
• oxygen fraction	. f_1.1-O_2	• oxygen fraction	f_1.2-O_2
• nitrogen fraction	• f_1.1-N_2	• nitrogen fraction	f_1.2-N_2
• carbon dioxide fraction	• f_1.1-CO_2	• carbon dioxide fraction	f_1.2-CO_2
• steam fraction	• f_1.1-H_2O	• steam fraction	f_1.2-H_2O

Other global quantities of interest:	
• global pressure drop coefficient	ξ
• Inlet section	A_in
• pressure drops at nominal conditions	Δp*
• pressure drop index	i_perd

Sizing procedure:

Knowing i_perd, A_in and Δp* or ξ the module can evaluate the other quantities using the relationship below:

$$\Delta p = \xi \cdot \rho_1 \cdot \frac{w_1^2}{2} \tag{4.38}$$

ρ_1 being the air density and w_1 the inlet air velocity.
Once that the pressure drops are evaluated, the module calculates the reference drop coefficient α* using:

$$\frac{\Delta p}{p_1} = \alpha \cdot \mu^2 \tag{4.39}$$

μ being the inlet corrected mass flow

$$\mu = \frac{m_1 \sqrt{T_1}}{p_1} \tag{4.40}$$

Off-design procedure:

The pressure drops are evaluated by means of the following equation:

$$\Delta p = \frac{\alpha^*}{f_{d\_p}} \cdot p_1 \cdot \left(\frac{m_1 \sqrt{T_1}}{p_1} \right)^2 \tag{4.41}$$

α* being the nominal pressure drop coefficient and $f_{d\text{-}p}$ being the fouling coefficient <=1.

4.3.2 Coil (Anti-icing)

The anti-icing module can be described as in figure 4.3:

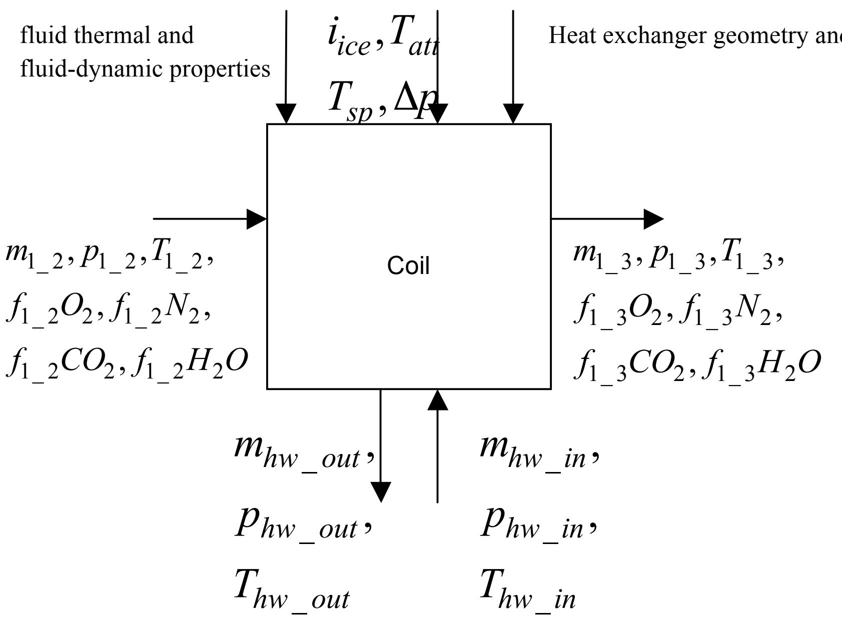


Fig. 4.3: anti-icing module scheme

A detailed description of the global quantities involved is given in the table below:

Inlet air:		Outlet air:	
• mass flow	m_1.2	• mass flow	m_1.3
• pressure	p_1.2	• pressure	p_1.3
• temperature	T_1.2	• temperature	T_1.3
• oxygen fraction	f_1.2-O_2	• oxygen fraction	f_1.3-O_2
• nitrogen fraction	f_1.2-N_2	• nitrogen fraction	f_1.3-N_2
• carbon dioxide fraction	f_1.2-CO_2	• carbon dioxide fraction	f_1.3-CO_2
• steam fraction	f_1.2-H_2O	• steam fraction	f_1.3-H_2O
Inlet hot water:		**Outlet hot water:**	
• mass flow	m_hw-in	• mass flow	m_hw-out
• pressure	p_hw-in	• pressure	p_hw-out
• temperature	T_hw-in	• temperature	T_hw-out

Sizing procedure

In the sizing procedure the following quantities have to be assigned as data of the problem:

- Inlet air mass flow, pressure, temperature and composition
- Desired outlet air temperature
- Inlet water mass flow, pressure and temperature
- Air and water velocities
- Architecture and geometric specifications

Fluid thermal and fluid-dynamic properties:	
Air	Water
• specific heat (constant pressure) c_p-air • specific heat (constant volume) c_v-air • dynamic viscosity μ_air • thermal conductivity λ_air	• specific heat (constant pressure) c_p-hw • specific heat (constant volume) c_v-hw • dynamic viscosity μ_hw • thermal conductivity λ_hw
Heat exchanger architecture and geometry:	
• tube internal diameter	d_i
• tube thickness	s_t
• tube conductivity	λ_t
• tube layout	i_maglia
• tube pitch	p_t
• heat exchange (physical phenomena)	i_scamb
• finning	i_alet
• ratio between finned and bared tube external surfaces	rap_est
• ratio between finned and bared tube internal surfaces	rap_int
• external finning efficiency	η_e
• internal finning efficiency	η_i
• air velocity	w_1°
• hot water velocity	w_hw-in
• thermal resistance due to the fouling effect on tube external surface	FSC_e
• thermal resistance due to the fouling effect on tube internal surface	FSC_i
• heat exchange global surface	S
• nominal pressure drops	Δp*
• anti-icing system activation index	i_ice

4.3.3 Filter

The filter pressure drop Δp_f at compressor inlet with that of actual mass flow at reference (when the filter is clean as new) is given as:

$$\Delta p_f = \Delta p_f^* \left[\frac{m_{air}}{m^*_{air}} \right]^{\alpha_f} K_f \tag{4.42}$$

m_{air}* being the clean reference filter air mass flow; Δp_f* standard filter pressure drop; α_f filter exponent, and filter fouling factor K_f.

4.3.4 Evaporative Cooler

The evaporative cooler module can be described as in figure 4.4:

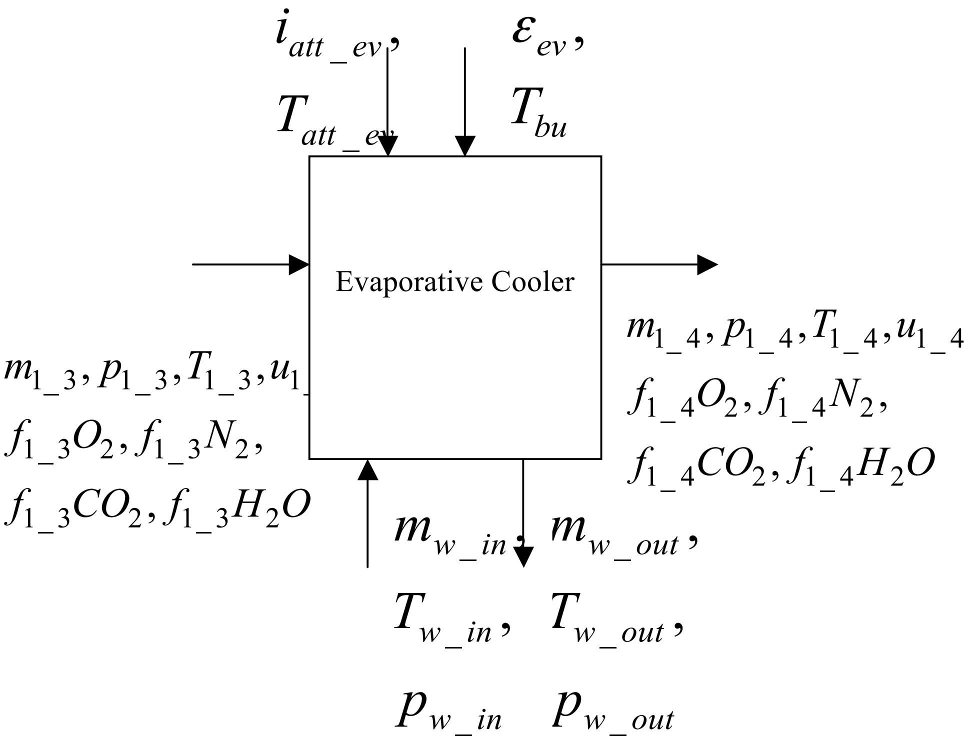


Fig. 4.4: Evaporative cooler module scheme

List of the variables involved is presented:

Inlet Air		Outlet Air	
• mass flow	m_1.3	• mass flow	m_1.4
• pressure	p_1.3	• pressure	p_1.4
• temperature	T_1.3	• temperature	T_1.4
• relative humidity	u_1.3	• relative humidity	u_1.4
• oxygen fraction	f_1.3-O_2	• oxygen fraction	f_1.4-O_2
• nitrogen fraction	f_1.3-N_2	• nitrogen fraction	f_1.4-N_2
• carbon dioxide fraction	f_1.3-CO_2	• carbon dioxide fraction	f_1.4-CO_2
• steam fraction	f_1.3-H_2O	• steam fraction	f_1.4-H_2O
Inlet water spray:		**Outlet water**	
• mass flow	m_w-in	• mass flow	m_w
• temperature	T_w-in		

List of data input to the module:

data	
• evaporative cooler efficiency	ε_ev
• wet bulb temperature	T_bu
• evaporative cooler activation temperature	T_att-ev
• reference pressure drops	Δp*
index	
evaporative cooler activation index	i_att-ev

T_1c is evaluated by the relation that takes the cooling effectiveness into consideration:

$$\varepsilon_{ev} = \frac{\left(T_{1\_b} - T_{1\_c}\right)}{\left(T_{1\_b} - T_{bu}\right)} \tag{4.43}$$

The model is based on the following rules of conservation:

- mass conservation:

$$m_{1\_b} + m_{w\_ev} = m_{1\_c} \tag{4.44}$$

m_w-ev being the evaporating mass flow

$$m_{w\_ev} = m_{w\_in} - m_{w\_out}\,; \qquad 4.45)$$

- Energy conservation:

$$m_{1\_b} \cdot h_{1\_b} + m_{w-in} \cdot h_{w-in} = m_{1\_c} \cdot h_{1\_c} + m_{w-out} \cdot h_{w-out} \tag{4.46}$$

h being the enthalpy.

Since for heat transfer processes the following relationships exist:

$$F(\varepsilon, NTU, c_{\min}, c_{\max}, architecture) = 0$$

$$NTU = \frac{U \cdot S}{c_{\min}}$$

and allow the off design calculations. The procedure is given in the paragraph 4.3.8.

4.3.5 Axial Compressor

The model uses the following equations:

- mass conservation

$$\rho_j A_j w_j \sin\beta_j - \rho_{j+1} A_{j+1} w_{j+1} \sin\beta_{j+1} = 0 \qquad (4.47)$$

ρ is the density, (A) is the annular section, (β) and (ω) are the fluid angle and blade relative velocity.

- energy conservation

$$h_{j+1} + 1/2\cdot(w_{j+1}^2 - u_{j+1}^2) = h_j + 1/2\cdot(w_j^2 - u_j^2) \qquad (4.48)$$

(h) and (u) represents the enthalpy and blade velocity.

- Momentum conservation

The work done by the rotor on the fluid, from the steady flow energy equation and momentum equation is:

$$L_E = u_{j+1} w_{j+1} \cos\beta_{j+1} - u_j w_j \cos\beta_j + u_{j+1}^2 - u_j^2 \qquad (4.49)$$

Across stator work done is zero.

- Correlation for calculating the losses

The total pressure losses in cascade is obtained with respect to the isentropic transformation. The total pressure at exit is correlated with the fluid condition at inlet;

$$p^0_{r,j+1} = p_j \left(\frac{T^0_{r,j}}{T_j}\right)^{c_p/R} - \frac{1}{2}\rho_j w_j^2 \omega \qquad (4.50)$$

where (T^0_r) is the total cascade temperature obtained from the conservation of energy given in equation 4.48:

$$T^0_r = T_j + \frac{w_j^2 - u_j^2}{2c_p} \qquad (4.51)$$

(ω) being the global coefficient of the total pressure loss defined as:

$$\omega = \frac{\Delta p^0_r}{\frac{1}{2}\rho_j w_j^2} \qquad (4.52)$$

The relation (4.52) is a generalized form of equation for the rotor and stator. In this equation the velocity corresponds to the relative velocity in rotor and absolute velocity in the stator. Of course the relative velocity of the stator blade is zero.

The model contains other relations which furnish thermodynamic properties.

The compression process for the complete stage is presented on a Mollier diagram in fig. 4.5, which is generalized to include the effect of irreversibility.

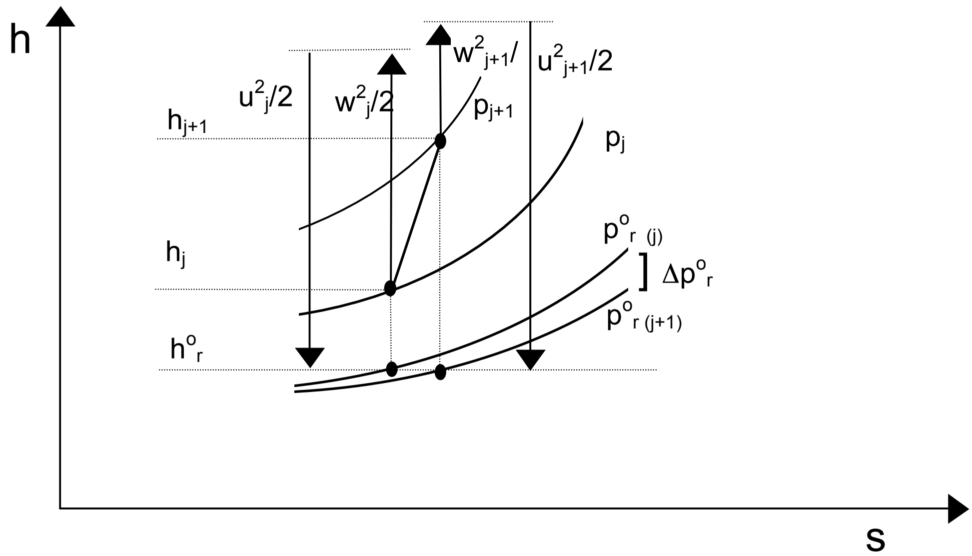


Fig. 4.5: h-s diagram flow of the fluid.

4.3.5.1 Model for Cycle calculation

The correlation adopted to calculate the compression process is expressed by thermodynamic differential equation,

$$\frac{dT}{T} = \frac{k-1}{k} \cdot \frac{1}{\eta_p} \cdot \frac{dp}{p} \tag{4.53}$$

with (dp) being polytropic efficiency, (k) ratio of specific heats which is calculated as a function of pressure, temperature, and gas compositions.
Calculation across compressor is conducted by dividing compressor inlet and exit pressure into small intervals (nint) of equal segments.
Given the compressor inlet conditions (i.e. inlet pressure, temperature and air mass flow) and exit pressure, and assuming compressor exit temperature (the exit temperature may be assumed equal to the inlet temperature), the actual compressor exit temperature can be obtained iteratively by integrating the polytropic transformation given in eq. 4.53, i.e.

$$\frac{T_{u,j}}{T_{i,j}} = \left(\frac{p_{u,j}}{p_{i,j}}\right)^{\frac{k_j-1}{k_j} \cdot \frac{1}{\eta_p}} \tag{4.54}$$

(Ti,j) and (Tu,j) respectively are the temperatures at inlet and exit of each interval j-th, (pu,j) and (pi,j)are the corresponding pressures, and (kj) represents the specific heat ratio correspond to the inlet condition. The calculation is performed simultaneously. The exit conditions from one interval are the inlet conditions to the next interval.

4.3.5.2 Sizing

Finite volume row by row lumped model is taken into consideration the through flow scheme is shown in figure 4.6.

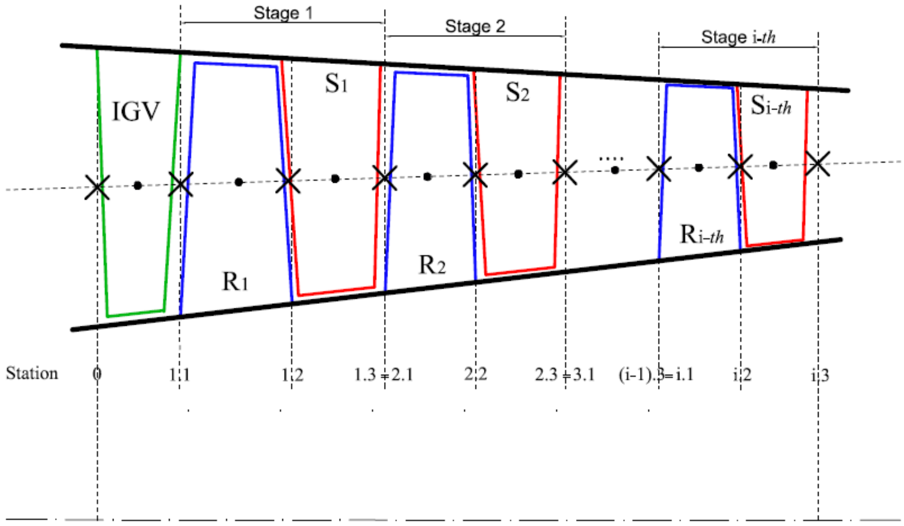


Fig.4.6: Generic compressor row by row scheme

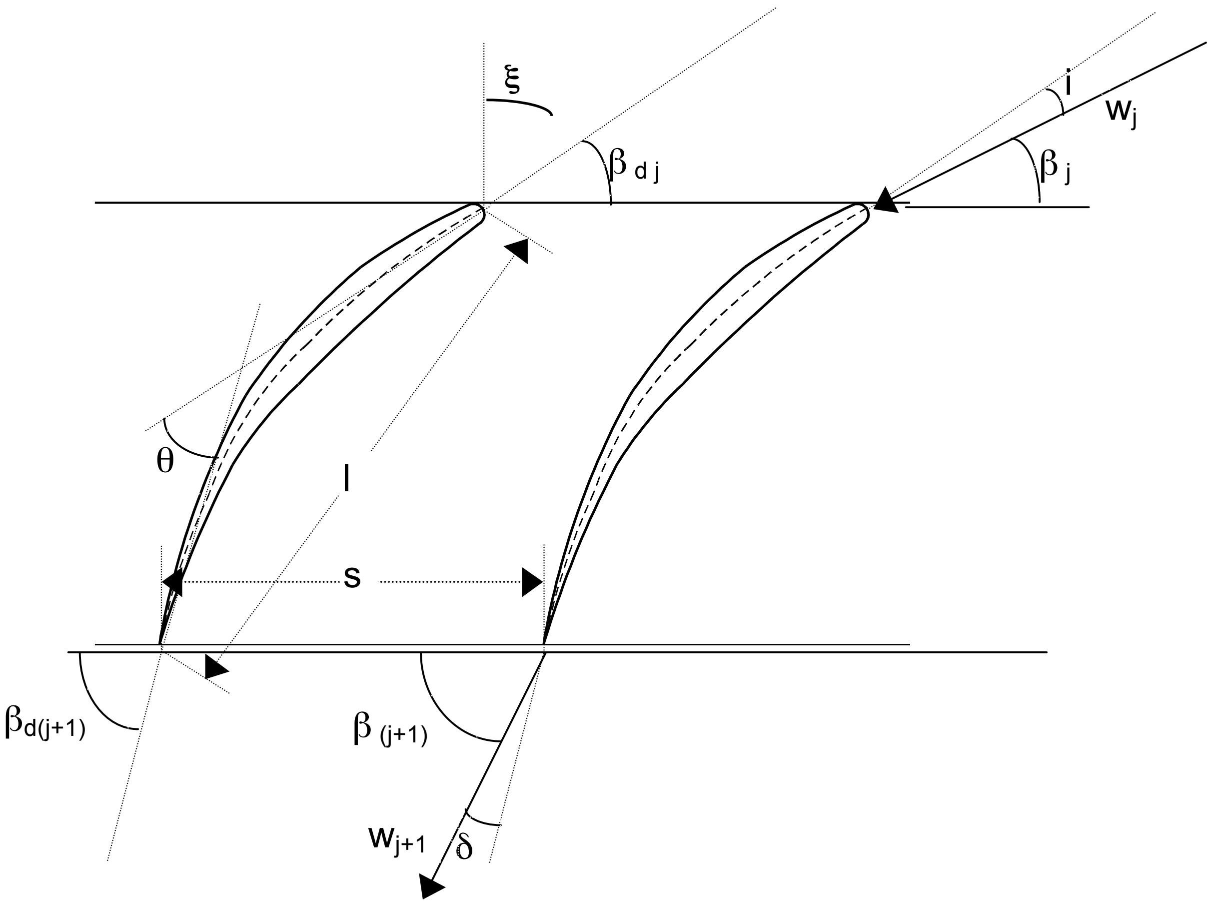


Fig. 4.7: Nomenclature adopted for cascade

The model is used to solve the inverse problem which determines the geometry and global parameters to characterize the compressor. Successively assigning these quantities the calculation is proceeded with the suitable number of working points that permit to establish the compressor map.
Figure 4.7 represents a cascade to cascade geometry flow considering the following equations written for the j^{th} and $j+1^{th}$ station with respect to inlet and exit to each cascade.
The problem is the sizing of a compressor which is for a given gas turbine (also for other applications) and equivalent to the real one whose real size are not known and there are not enough information's (from the manufacturer). The compressor have been divided in to stationary and moving rows as shown in figure 4.5. The model of a finite volume lumped row by row steady flow.
The geometry data of compressor are provided by the manufacturer. If these data are not available they are calculated by a preliminary solution of an inverse (sizing) problem using RO3 methodology.
The empirical models of the flow features and performance (deviation, loss, etc.) reported in this section is one of the option that can be chosen. In fact there are various models that can be selected (Anely and Medison, Howell, etc.).

The flow and working fluids quantities of compressor at nominal conditions given as input quantities are:

- air mass flow; kg/s
- rotational speed; rpm
- inlet pressure (p_{in}); bar
- inlet temperature (T_{in}); °C
- pressure ratio (β);
- number of stages (N_{stage});
- number of stator with varying geometry;
- inlet air mass compositions; %
- index for the presence of inlet and outlet guide vanes [VIGV and OGV]; (if 0 there is no VIGV, if 1 There exists an VIGV).
- index for velocity; (if 0 then speed is constant, if 1 speed is varying, then the program read the maximum and minimum velocities);
- constructive efficiency (etacons);
- optimum reaction coefficient (optcoef);
- maximum permissible peripheral velocity ($u1_{max}$);
- maximum permissible Mach number (Ma_{max});
- design total exit temperature ($t3_{des}$);

Variables introduce in the program are:

- blade heights for each station (i.e. stator and rotor);
- peripheral blade velocity (u_l) corresponding to the 1st station;
- ratio of actual losses to that of calculated losses (fr).

the geometrical quantities are unknowns.
The compressor model calculates thermodynamic and fluid dynamic quantities at the exit of each row (which are the entrance to the next row), the whole machine calculate the stacking of the contribution of various rows. The model use mass, energy, continuity, and 2^{nd} law equation, empirical correlation's for losses, and deviation calculations.
Consider one stage of the compressor consisting of a fixed and moving row (i.e. stator and rotor). The inlet flow to the first row (stator or VIGV) is indicated by station 1. The exit quantities from first row which are the inlet quantities to the next row is denoted be station 2, and the exit of moving row is shown by station 3.
The thermodynamic quantities and geometric data using row-by-row sizing of the compressor are obtained from the following relations.
First of all experience has indicated that the axial velocity should remain constant at the design point. Thus

$$w_j\,sin\beta_j\ -\ w_{j+1}\,cos\beta_{j+1} = 0 \qquad (4.55)$$

and

$$C_j\,sin\alpha_j\ -\ C_{j+1}\,sin\alpha_{j+1} = 0 \qquad (4.56)$$

(w_j) and (w_{j+1}) being the relative velocities at inlet and exit. (Cj) and (Cj+1) are the absolute velocities. (α) and (β) are the relative and absolute angles as shown in the triangular velocity diagram (see fig. 4.8).

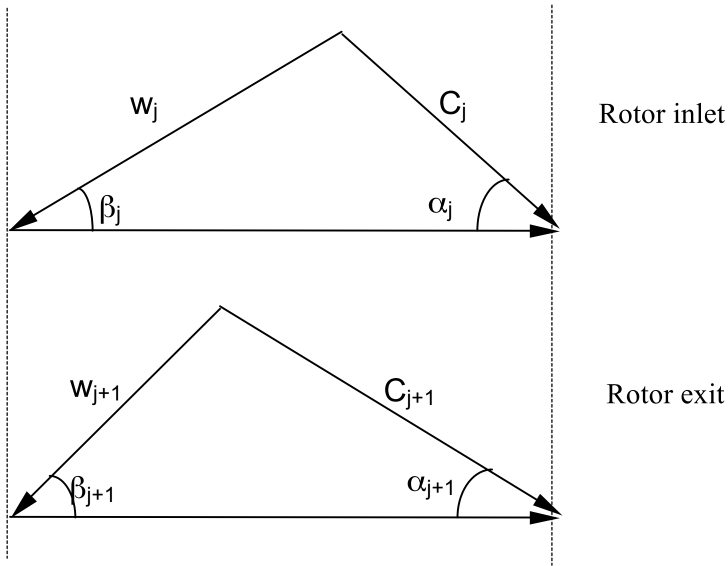


Fig. 4.8: Rotor triangular velocity diagrams

- Degree of reaction

The degree of reaction provides the extent to which the rotor contributes to the overall static pressure rise in the stage, and is given as;

$$R = f\,(w_{u\infty},\ u) \qquad (4.57)$$

being

$$w_{u\infty} = (w_j\,cos\beta_j + w_{j+1}\,cos\beta_{j+1})/2 \qquad (4.58)$$

and

$$u = (u_{j+1} + u_j)/2 \qquad 4.59)$$

j and j+1 being the inlet and exit conditions from each moving blade row.

- Flow coefficient at the entrance of each stage

From continuity equation, assuming incompressibility, yields;

$$w_j\ cos\beta_j = c_{a,j} = w_{j+1}\ cos\beta_{j+1} = c_{a,j+1} = c_a \qquad (4.60)$$

the flow coefficient is

$$\Phi_i = f\,(c_{aj},\ u) = w_j sin\beta_j/u \qquad (4.61)$$

with (i) indicating each stage and (j) corresponds to the inlet condition. (c_{aj}) is the axial velocity and (u_j) is the blade moving row velocity. (Φ_i) is being determined.
Given the maximum stage efficiency and degree of reaction the flow coefficient can be written as follow;

$$\Phi_i = A\sqrt{R^2 - R + 0.5} \tag{4.62}$$

where (A) is the optimum coefficient for calculating the flow coefficient, denoted by optcoef in input data.

- Total and static temperatures and pressures

the static temperature at the rotor inlet is;

$$T_j = T_{in} - \frac{C_j^2}{2C_p} \tag{4.63}$$

the static pressure at the rotor inlet is;

$$p_j = p_{in}\left(\frac{T_j}{T_{in}}\right)^{Cp/R} \tag{4.64}$$

the total relative temperature at the rotor inlet is;

$$T^{\circ}_{rj} = T_j + \frac{w_j^2 - u_j^2}{2C_p} \tag{4.65}$$

the relative total pressure is;

$$p^{\circ}_{rj} = p_j\left(\frac{T^{\circ}_{rj}}{T_j}\right)^{Cp/R} \tag{4.66}$$

the total relative temperature at the rotor exit is equal to that of the rotor inlet, therefore, the static temperature is given by;

$$T_{j+1} = T^{\circ}_{rj} - \frac{w_{j+1}^2 - u_{j+1}^2}{2C_p} \tag{4.67}$$

- Aspect ratio

The ratio of height (H) to chord (l) is called as aspect ratio and is defined as:

$$AR = H/l \tag{4.68}$$

- Solidity

The ratio of chord (l) to pitch (s) is called solidity and is defines as:

$$\sigma = l/s \tag{4.69}$$

- Incidence angle

The optimum incidence angle at inlet to each blade is

$i_d = i_o + n\theta - 0.5$ (4.70)

where (θ) is the blade chamber angle, (i_0) and (n) are the indexes obtained as a function of flow inlet angle reported in figure 4.8

- <u>chamber angle (θ)</u>

Is the difference between the blade inlet angle and blade exit angle (see figure 4.6).

- <u>Mach number</u>

It is calculated as

$$Ma = \frac{w_1}{\sqrt{\gamma R T_1}} \tag{4.71}$$

w_1 being the relative velocity and T_1 is the temperature at the inlet to each stage.

- <u>Stream deviation</u>

The deviation angle is the shifting between the fluid angle and blade angle at exit of the blade, and is expressed as;

$\delta_d = \delta_0 + m\phi - 0.5\Delta$ (4.72)

with δ_0, m, Δ are being calculated as a function of the flow inlet angle β_j as shown in figure 4.9.

A contribution of losses are taken into account evaluating for each row related to profile losses, annulus losses, and secondary losses, and are defined as follow:

- <u>Annulus losses</u>

Taking into account the losses due to drag on the annulus surface, the corresponding drag coefficient is given by;

$$C_{DA} = 0.02\,(s/H) = 0.02\,\frac{\sigma}{(H/l)} \tag{4.73}$$

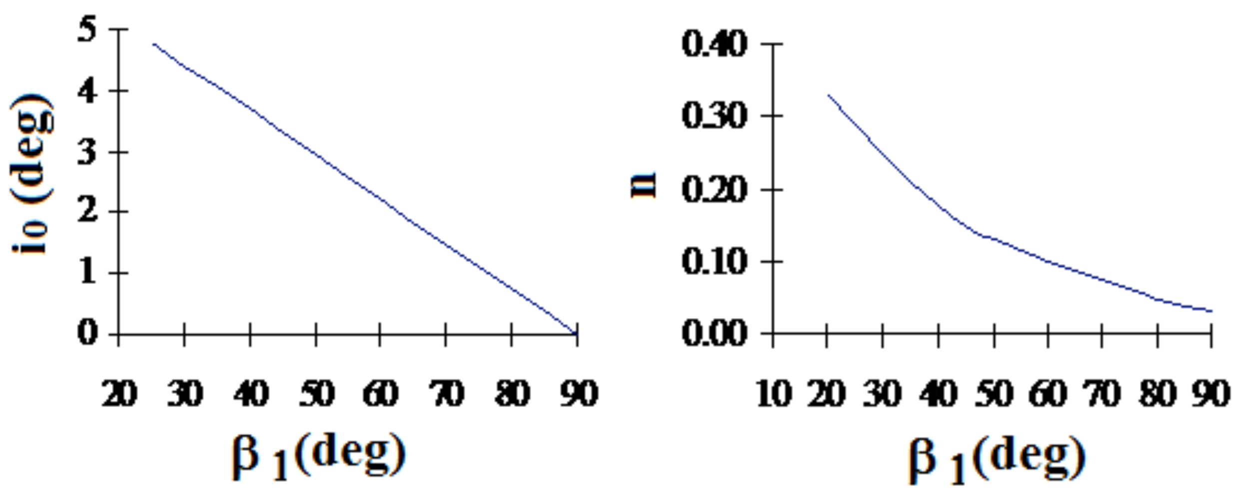


Fig. 4.9: Correlation used to determine the optimum incidence.

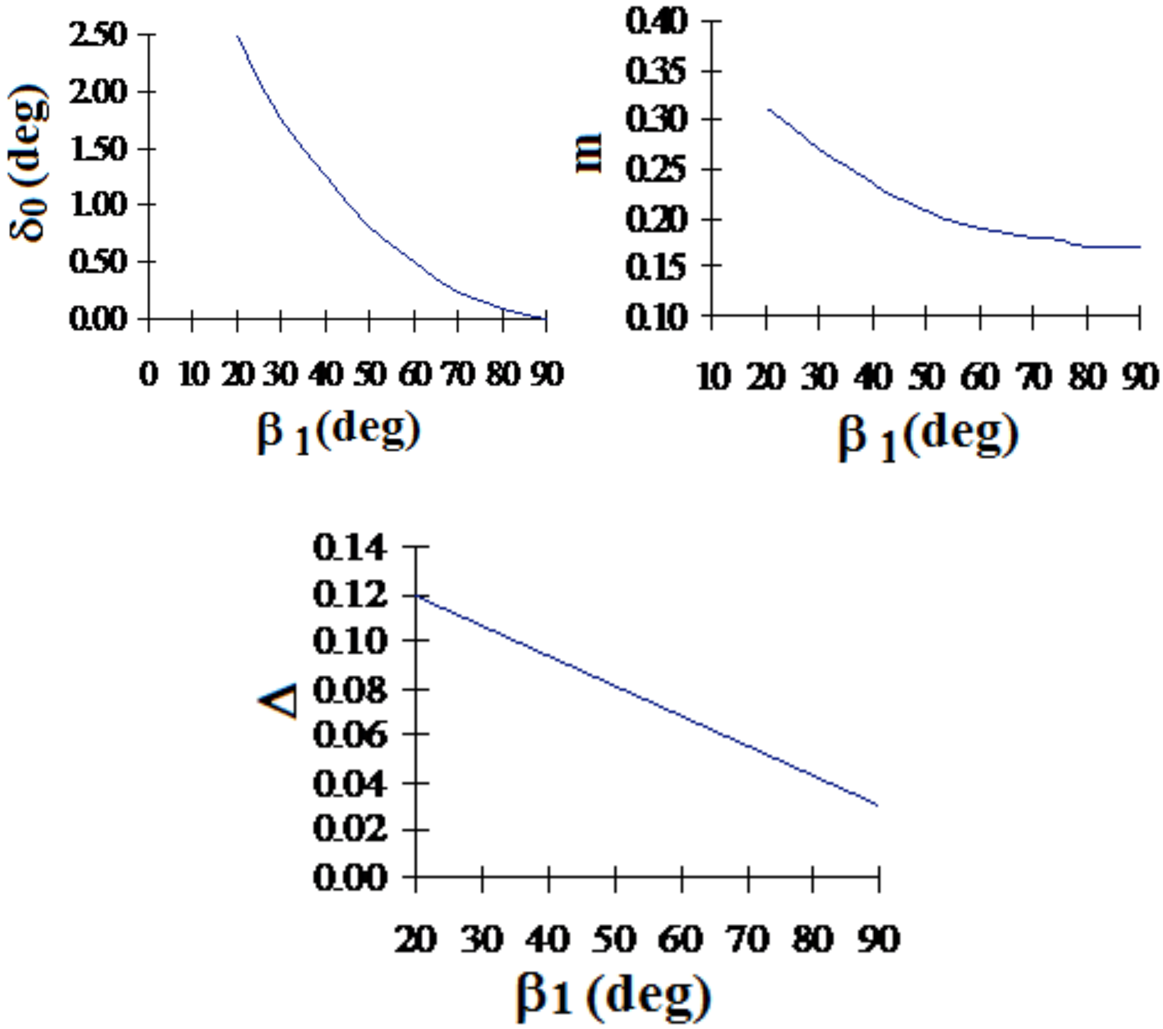


Fig. 4.10: correlations used to calculate deviation

- <u>Profile losses</u>

The relative coefficient for total pressure losses ω_{pd} is given as a function of diffusion factor (D);

$$D = 1 - \frac{w_{j+1}}{w_j} + \frac{w_j \cos\beta_j - w_{j+1}\cos\beta_{j+1}}{2\sigma w_j} \tag{4.74}$$

by means of correlation given in figure 4.11, the parameter f(D) is obtained and accordingly the total pressure loss ω_{pd} is being calculated from eq. 4.75.

$$f(D) = \frac{\omega_{pd}\sin^3\beta_{j+1}}{2\sigma} \tag{4.75}$$

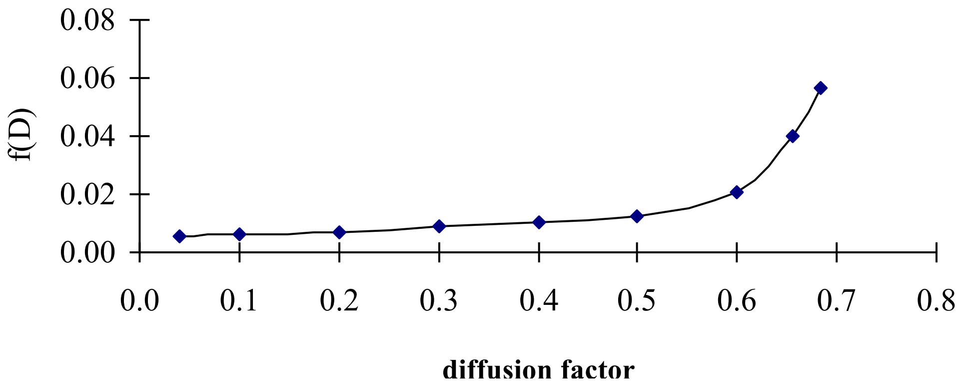


Fig. 4.11: Correlation used to calculate the profile losses.

- Secondary losses

Arises from secondary flows which are always present when a wall boundary layer is turned through an angle by an adjacent curved surface, and is expressed as;

$$C_{DS} = 0.018\, C_L^2 \tag{4.76}$$

C_L being the lift coefficient and is given as:

$$C_L = (2/\sigma) \sin \beta_m (\cot\beta_j + \cot\beta_{j+1}) \tag{4.77}$$

and angle (β_m) is the:

$$\beta_m = \tan [0.5 (\cot\beta_j + \cot\beta_{j+1})] \tag{4.78}$$

- Global coefficient

The global coefficient loss is defined by ω:

$$\omega = \frac{\Delta p_r^0}{\frac{\rho_j w_j^2}{2}} = (C_{DA} + C_{DS}) \frac{\sigma \sin^2\beta_j}{\sin^2\beta_m} + \omega_{pd} \tag{4.79}$$

j referring to the inlet condition to each cascade.

Once calculated the losses which defines:

- incidence (i_r)
- camber angle (θ_r)
- stagger angle (ξ_r)
- thickness ratio (TR_r)
- aspect ratio (AR_r)
- solidity (σ_r)
- Mach number (Ma_r)

and other details such as clearance, surface roughness, and so on. typical empirical functions are assumed according to the level of technology. Thus the loss coefficient for the rotor is defined as:

$$\overline{\omega}_r = \overline{\omega}_r (ir, \theta r, \xi r, TRr, ARr, \sigma r, Mar) \tag{4.80}$$

Therefore the total rotor exit relative pressure is:

$$p^{\circ}_{rj+1} = p^{\circ}_{j} - \frac{1}{2}\overline{\omega}_{r}\rho_{j}w^{2}_{j+1} \tag{4.81}$$

and the static pressure is:

$$p_{j+1} = p^{\circ}_{j+1}\left(\frac{T_{j+1}}{T^{\circ}_{rj}}\right)^{Cp/R} \tag{4.82}$$

The blade height can be establish from the continuity equation. Therefore for each station of one stage, we have:

$$p_1 = R\,T_1\,\rho_1 \tag{4.83}$$

$$\rho_1\,\pi\,(d_{ref}\,\chi_1\,H_1 + H_1^2)w_1 sin\beta_1 = m \tag{4.84}$$

where area (A) is given by;

$$A = \frac{\pi}{4}\left(D^2{}_{tip} - D^2{}_{hub}\right)$$

$$A = \frac{\pi}{4}\left[(d_{ref} + 2H_1)^2 - d^2{}_{ref}\right]$$

$$A = \frac{\pi}{4}\left[d_{ref}\,H_1 + H_1{}^2\right]$$

and for station 2

$$p_2 = R\,T_2\,\rho_2 \tag{4.85}$$

$$\rho_2\,\pi\,(d_{ref}\,\chi_2\,H_2 + H_2^2)w_2 sin\beta_2 = m \tag{4.86}$$

similarly for station 3

$$p_3 = R\,T_3\,\rho_3 \tag{4.87}$$

$$\rho_3\,\pi\,(d_{ref}\,\chi_3\,H_3 + H_3^2)C_3 sin\alpha_3 = m \tag{4.88}$$

To go at the stator exit the total temperature at the stator inlet is;

$$T^{\circ}_{j+1} = T^{\circ}_{j} + u_{j+1}{}^2 - u_j{}^2 - u_{j+1}\,w_{j+1}cos\beta_{j+1} + u_j\,w_j\,cos\beta_j \tag{4.89}$$

the total pressure is

$$p^{\circ}{}_{j+1} = p_{j+1}\left(\frac{T^{\circ}{}_{j+1}}{T_{j+1}}\right)^{Cp/R} \qquad (4.90)$$

The loss coefficient for the stator is defined as:

$$\overline{\omega}_s = \overline{\omega}_s(i_s, \theta_s, \xi_s, TR_s, AR_s, \sigma_s, Ma_s) \qquad (4.91)$$

- incidence (i_s)
- camber angle (θ_s)
- stagger angle (ξ_s)
- thickness ratio (TR_s)
- aspect ratio (AR_s)
- solidity (σ_s)
- Mach number (Ma_s)

The total temperature at station 3 is;

$$T^{\circ}{}_3 = T^{\circ}{}_2 \qquad (4.92)$$

$$T_3 = T^{\circ}{}_2 - \frac{C_3{}^2}{2C_p} \qquad (4.93)$$

Therefore the total rotor exit pressure at station 3 is given by:

$$p_3 = p^{\circ}{}_2 - \frac{1}{2}\overline{\omega}_s \rho_2 C^2{}_2 \qquad (4.94)$$

and

$$p_3 = p^{\circ}{}_3\left(\frac{T_3}{T^{\circ}{}_3}\right)^{Cp/R} \qquad (4.95)$$

The blade angle for the rotor is given by:

$$\beta_{b1} + \theta_r - \beta_{b2} = 0 \qquad (4.96)$$

$$\beta_1 + i_r - \beta_{b1} = 0 \qquad (4.97)$$

$$\beta_2 + \delta_r - \beta_{b2} = 0 \qquad (4.98)$$

(β_{b1}) and (β_{b2}) being the rotor blade inlet and exit angles, (θ_r) is the rotor camber angle, (i_r) is the rotor incidence angle, and (δ_r) is the rotor deflection angle.

Similarly the stator blade angles are given by:

$$\alpha_{b2} + \theta_s - \alpha_{b3} = 0 \qquad (4.99)$$

$$\alpha_2 + i_s - \alpha_{b2} = 0 \qquad (4.100)$$

$\alpha_3 + \delta_s - \alpha_{b3} = 0$ (4.101)

where (α_{b1}) and (α_{b2}) are the stator blade inlet and exit angles, (θ_s) is the stator camber angle, (i_r) is the stator incidence angle, and (δ_s) is the stator deflection angle.

If the blade camber line is circular type, then for rotor

$\xi_r = (\beta_{b_1} + \beta_{b2})/2$ (4.102)

for stator

$\xi_s = (\alpha_{b_1} + \alpha_{b2})/2$ (4.103)

The peripheral velocities are:

$u_1 = \pi\, n\, (d_{ref}\, \chi_1 + H_1)/60$ (4.104)

$u_2 = \pi\, n\, (d_{ref}\, \chi_2 + H_2)/60$ (4.105)

(u_1) and (u_2) being the inlet and exit peripheral velocities of the moving blades, (H_1) and (H_2) being the blade heights, (n) being the rotational speed, (d_{ref}) being the reference hub diameter ratio, (χ_1) and (χ_2) are the diameter ratios at each station.
Note that

$\omega = \pi n/30$
$u = \omega\, D = \pi nR/60 = \pi n\, (dref\ \chi 1 + H1)/60$
(D) being the mean diameter, (R) being the mean radius and (n) rotational speed

This procedure is repeated for N-number of stages. The exit quantities from one stage are the inlet quantities to the next stage.

- <u>Mechanical Power</u>

The mechanical power is calculated by;

$P^*_{mp} = (1 - \eta_m)\, P_m$ (4.106)

with $P_m = P_i - P_u$ taking into account the mechanical power. Of course this proceed to determine the constant

$K_m = P^*_{mp}/n^*$ (4.107)

with n^* reporting the rotational velocity of machine at nominal condition of air mass compositions.

4.3.5.3 Part load analysis

The schematic diagram of the model is shown in figure 4.12

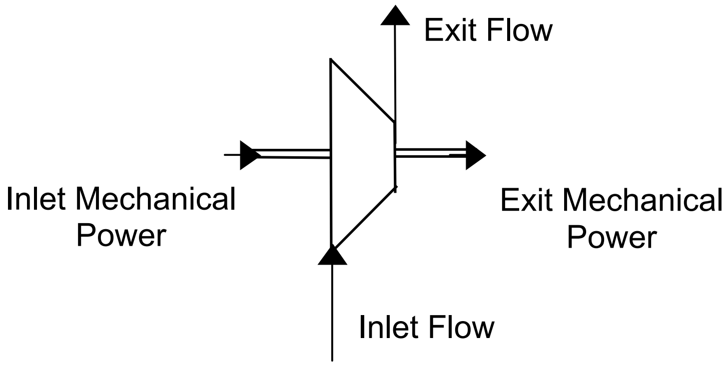


Fig. 4.12: Schematic model of axial compressor

The corresponding inlet and exit variables and power across the compressor are defined as:

Inlet fluid	m	mass flow	[kg/s]
	p_i	pressure	[MPa]
	T_i	temperature	[°C]
	composition (mass fractions O_2, N_2, CO_2, H_2O)		
Exit fluid	m	mass flow	[kg/s]
	p_u	pressure	[MPa]
	T_u	temperature	[°C]
	compositions (mass fractions O_2, N_2, CO_2, H_2O)		
Inlet mechanical power	P_{mi}	mechanical power	[MW]
	n	rotational s	[rpm]
Exit mechanical power	P_{mu}	mechanical power	[MW]
	n	rotational velocity	[rpm]

Compressor geometric data for each stage are given as a input are:

- VIGV absolute flow angles at inlet and exit, camber angle, solidity (of course these quantities are read if VIGV exists);
- stator absolute angles at inlet and exit [°];
- rotor relative angles at inlet and exit [°];
- diameter ratios [m];
- height at each station [m];
- area at each station [m^2];
- stator and rotor camber angles [°];
- stator and rotor stagger angles [°];
- stator and rotor solidity;
- stator and rotor aspect ratio;
- stator and rotor maximum thickness ratio;
- stator and rotor pressure losses;
- stator and rotor inlet and exit blade angles [°];
- stator and rotor incidence angle [°];

The different operating points are determined giving the values of:

- compressor inlet pressure and temperature;
- rotational speed; and
- exit pressure

At various junctions between the adjoining modules mechanical power and rotational speed are defined as variables.
the axial compressor model is a constant mass flow model: the code assign automatically the same value of mass flow at compressor inlet and outlet. conservation of mass is automatically satisfied.
In the same way the code assign the same name to the variables whose values do not change during model calculation (such as fluid compositions, rotational speed, etc.).

The energy conservation is written as:

$$m\,(h_u - h_i) = P_{mi} - P_{mu} - P_{mp} \qquad (4.108)$$

(h_u) and (h_i) being the enthalpy of fluid at exit and inlet, calculated by means of state equation as a function of pressure, temperature, and compositions. P_{mp} is the mechanical lost power calculated by means of an assigned coefficient.

Compressor inlet pressure (p_i) is expressed as:

$$p_i = p_a \cdot (1 - \Delta p_f) \qquad (4.109)$$

Δp_f being the inlet duct pressure loss, p_a being the atmospheric pressure.

For the off-design condition following relations are used:

$$F(\beta, m, t_i, p_i, n, \alpha) = 0 \qquad (4.110)$$

and

$$F(\beta, \eta, t_i, p_i, n, \alpha) = 0 \qquad (4.111)$$

(p_i) and (T_i) being the inlet conditions to the compressor, $\beta = p_u/p_i$ represents the compressor pressure ratio, (η) is the adiabatic efficiency. (n) is the rotational velocity and (α) being the parameter for the opening of the VIGV for stator cascade with variable geometry. Care should be taken for 3 different cases:

a. *compressor with fixed geometry:* the calculation are obtained for different values of rotational speed. The upper and lower limits have been imposed for compressor rotational speed;
b. *compressor with variable geometry and fixed rotational speed*: are calculated for different values of α;
c. *compressor with variable geometry and variable rotational speed:* a role is taken that correlates the opening of the moving vanes with the rotational velocity. Different values of rotational speed (n) for a field given by the user.

for each value of the parameters (n) and (α) the calculation is done for different machine fluid inlet temperature. The maximum and minimum values of the temperature are given by the user.
On each curve surge condition is assumed when the row is stalled. Stall limits for positive and negative incidence angle are found using the Mellor's diagram (Horlock, 1858). This correlation is available in the subroutine MELLORI.
If in the stage there isn't pressure increment choking condition is assumed.
The only change in the model with respect to inverse problem is that of the calculation of the profile losses and of the flow deviation.

For the profile losses a parabolic variation of the relative coefficient of the total pressure losses is assumed, which is given as:

$$\omega_p = \omega_{pd}(0.833s^2 + 0.167s + 1.0) \quad (4.112)$$

with

$$s = \frac{i}{\beta_{jd} - \beta_{jc}} \quad \text{for } i<0 \quad (4.113)$$

and

$$s = \frac{i}{\beta_{jd} - \beta_{js}} \quad \text{for } i>0 \quad (4.114)$$

ω_{pd} being the coefficient of total losses at the nominal condition (determined from the solution of inverse problem), (β_{jd}) is the constructive angle of the blade at leading edge. (β_{js}) and (β_{jc}) are the fluid angles when there are conditions of positive and negative stalls.

The flow deflection $\varepsilon = \beta_{j+1}-\beta_j$ through the cascade is calculated as a function of incidence with a curve given in fig. 8. This curve is normalized with the value of i_d and ε_d i.e. incidence and deflection of reference points. With the flow deflection the calculation of the deviation and the flow angle at the exit is faster.

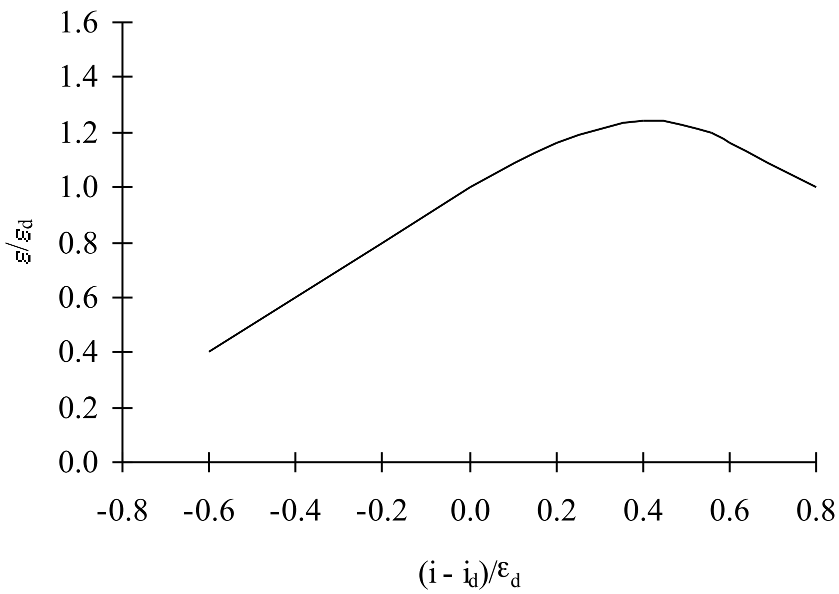


Fig. 4.12: correlation used for calculating the deflection

Enthalpy of air at compressor inlet and exit are:

$$h_i = F(p_i, T_i, [XM]_i) \quad (4.115)$$

$$h_u = F(p_u, T_u, [XM]_u) \quad (4.116)$$

$[XM]_{air}$ being the fractions of air mass compositions.

4.3.6 Combustion chamber (C.C.)

The combustion chamber is treated as a system producing a certain design point pressure drop with energy, mass and momentum conservation and with chemical reaction.
Chemical reactions are assumed to be at equilibrium. Fuel composition is given in term of fractions of C, S, H_2, O_2, N_2, and H_2O (vapour). Stoichiometric equations have been implemented to account for oxidation of above components are:

$$C + O_2 = CO_2 \tag{4.117}$$

$$S + O_2 = SO_2 \tag{4.118}$$

$$2H_2 + O_2 = H_2O \tag{4.119}$$

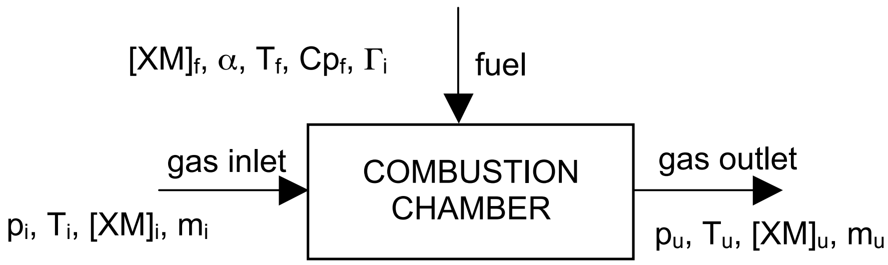


Fig. 4.13: Combustion Chamber Specifications

Consider a combustion chamber (C.C.), as shown in figure 4.13 with the following reference air and fuel specifications entering into the combustion chamber:

- mass flow m_i [kg/s];
- inlet pressure p_i [kPa];
- inlet temperature T_i [K];
- exit pressure p_u [kPa];
- mass fractions of fuel compositions $[XM]_f$ consisting C, S, H_2, O_2, N_2, and H_2O [%];
- fuel lower heating value Γ_i [kJ/kg];
- fuel temperature T_f [K];
- fuel specific heat at constant pressure Cp_f [kJ/kg K];
- combustion chamber efficiency η_b [%];
- combustion chamber pressure drop ($\Delta p_{c.c.}$)

The decision variables are:

- gas mass flow at inlet m_i [kg/s];
- gas inlet pressure p_i [kPa];
- gas inlet temperature T_i [°C];
- fractions of air mass compositions at inlet $[XM]_i$, consisting O_2, N_2, CO_2, and H_2O [%].
- fuel mass flow m_f [kg/s];
- fuel inlet pressure p_f [kPa];
- fuel inlet temperature T_f [°C];
- fractions of fuel mass compositions at inlet $[XM]_i$, consisting O_2, S, N_2, and H_2O [%].
- water/steam mass flow m_w [kg/s];

- water/steam inlet pressure p_w [kPa];
- water/steam inlet temperature T_w [°C];
- water/steam inlet enthalpy h_w [kJ/kg];
- gas mass flow at exit m_u [kg/s];
- gas exit pressure p_u [kPa];
- gas exit temperature T_u [K];
- fractions of gas mass compositions at exit $[XM]_i$ consisting O_2, N_2, CO_2, and H_2O [%].

This module evaluates the following quantities

- thermal load;
- pressure loss;
- energy losses;
- product gases compositions;
- combustion chamber efficiency;
- availability costs and operational costs.

product of gas compositions $[FB]_{gas}$ burning 1 kg of fuel:

$$[FB]_{gas} = f([XM]_u, [XM]_i) \qquad (4.120)$$

fraction of mass compositions of the product gases $[XM]_u$ at C.C. exit:

$$[XM]_u = f([XM]_i, [XM]_f , \alpha) \qquad (4.121)$$

A constraint is implemented to check that the actual air-fuel ratio is not less than the stoichiometric air-fuel ratio.
enthalpy of air at C.C. inlet

$$h_i = f(p_i, T_i, [XM]_i) \qquad (4.122)$$

The module consists of:

- <u>mass conservation</u>

$$m_u = m_i + m_f + m_w \qquad (4.123)$$

- <u>energy conservation</u>

$$m_u\, h_{uc} = m_a\, h_a + m_f (Cp_f\, T_f + \eta_b\, \Gamma_i) + m_w\, h_w \qquad (4.124)$$

The product gases temperature (T_{gas}) is achieved using equality constraint that has to satisfy the state point enthalpy of gases at combustion chamber i.e.

$$h_u = f(p_u, T_u, [XM]_u) \qquad (4.125)$$

The pressure drop across combustion chamber ($\Delta p_{c.c.}$) occurs mainly due to the frictional pressure loos. Thus combustion chamber exit pressure is established as:

$$p_u = p_i (1 - Kp_u) \qquad (4.126)$$

($Kp_{u.}$) being the combustion chamber fouling factor and is given by

$$K_{p_u} = K_{fp}\left[\frac{m_i\sqrt{T_i}}{p_i}\right]^2 \tag{4.127}$$

(K_{fp}) is a constant and its value is determined based on the pressure loss and gas conditions at combustion chamber inlet with respect to nominal point.
For the off-design calculation the thermal losses are established as a function of pressure and temperature difference between two streams in combustion chamber using generalized relationships with respect to the reference data. Figure 4.14 shows the curves for off-design calculation of combustion efficiency versus the temperature difference between two streams varying inlet pressures:

$$\eta_b = f(p_i, \Delta T_b, CRF) \tag{4.128}$$

(p_i) being the combustion chamber inlet pressure, (ΔT_b) the difference between the two stream temperatures in across combustion chamber, and (CRF) is the combustion chamber correction factor and is obtained from the design calculation, respectively.

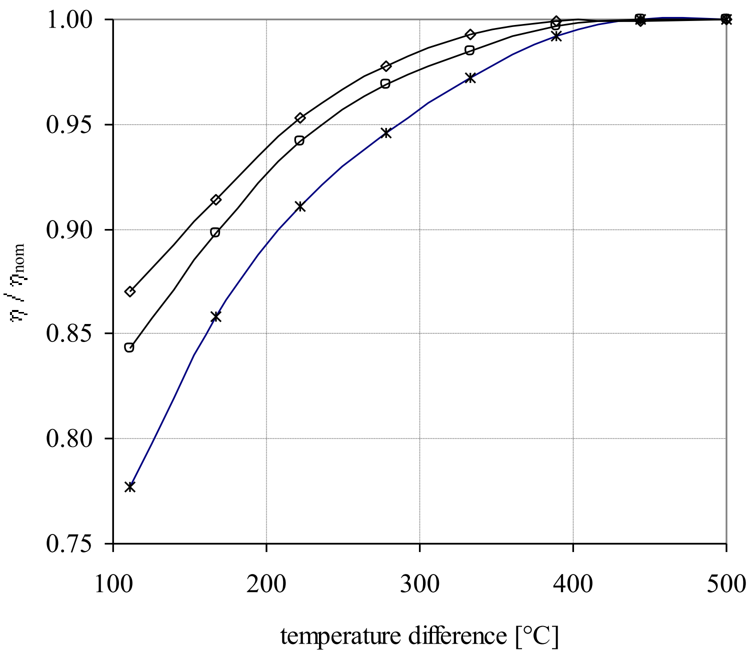


Fig. 4.14: Generalized curve to determine combustion chamber efficiency.

4.3.7 Gas Expander

Figure 4.15 shows a four-stage gas expander. In figure. 4.16 the profile of the blade are reported. The quantities of interested related to blade and flow features are also reported.
The model describes the flow through a blade cascade (stationary or rotating). the model takes into account the air mass flows. Consider one stage of gas expander in figure 4.15, for each station 1, 2, and 3 the following variables are assumed:

- pressure, temperature, and mass flow of the gas;
- mass flow of coolant;
- fluid angles.

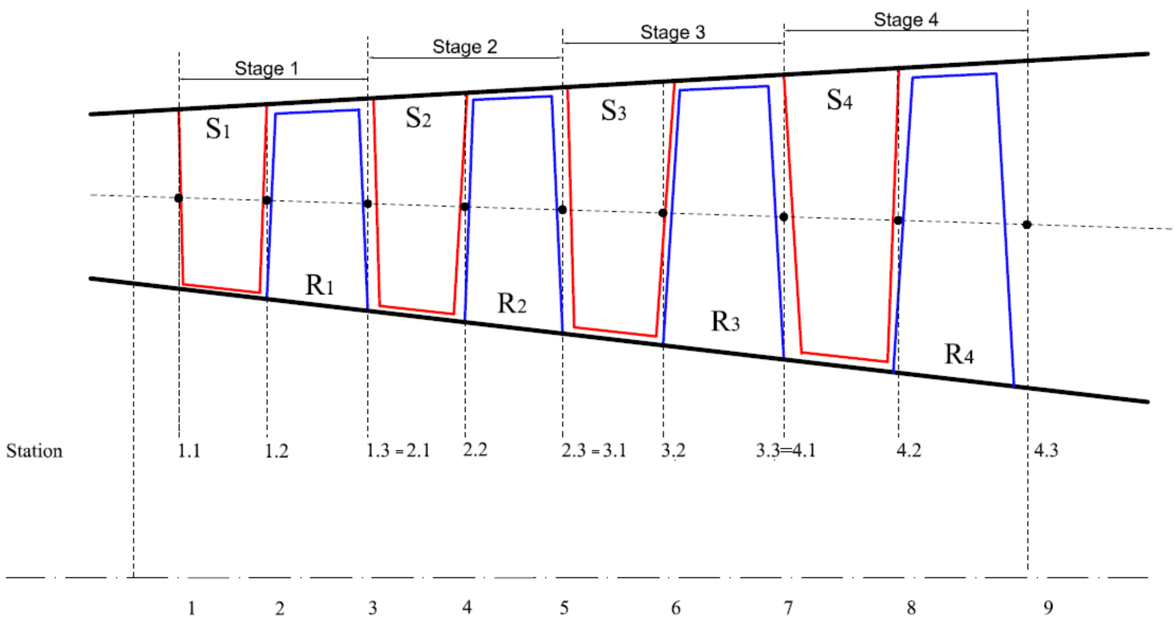


Fig. 4.15: Generic gas expander row by row scheme

The expansion of both gas stream and coolant are studied separately considering the evaluation of two fluids and then mixing them as shown in figure 4.17. The quantities involved in the calculation corresponding to inlet and exit blades are shown. For the sake of generality the following refers to a mixing blade row. The expansion of the main gas stream and coolant flows are described in fig. 4.18 on a h-s diagram, showing the thermodynamic statuses of the flow during the process.

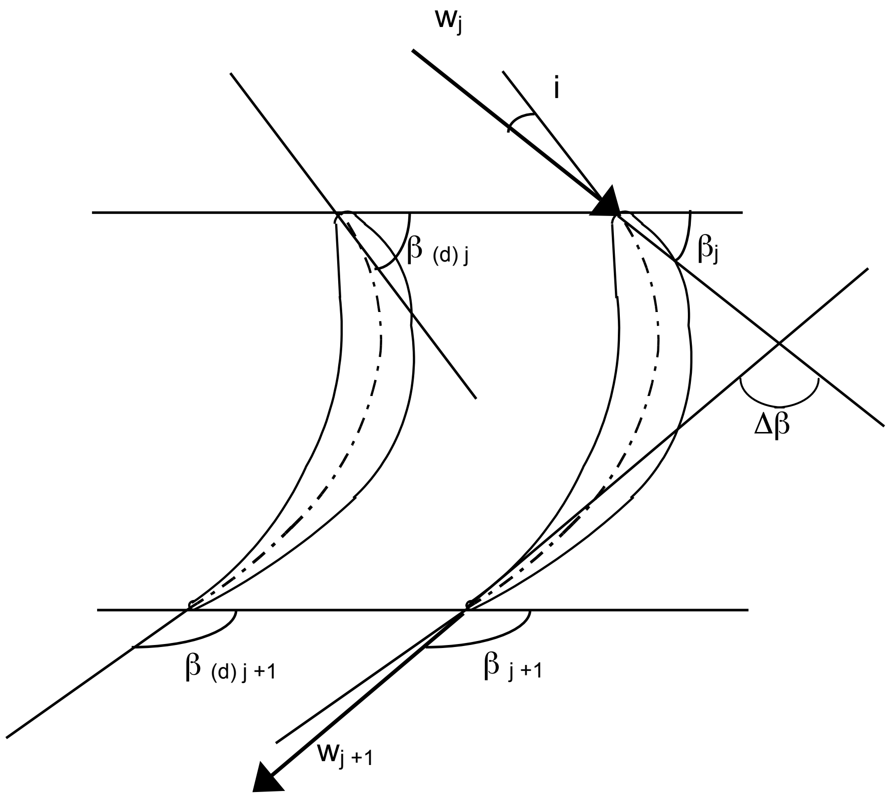


Fig. 4.16 Nomenclature adopted for the cascade.

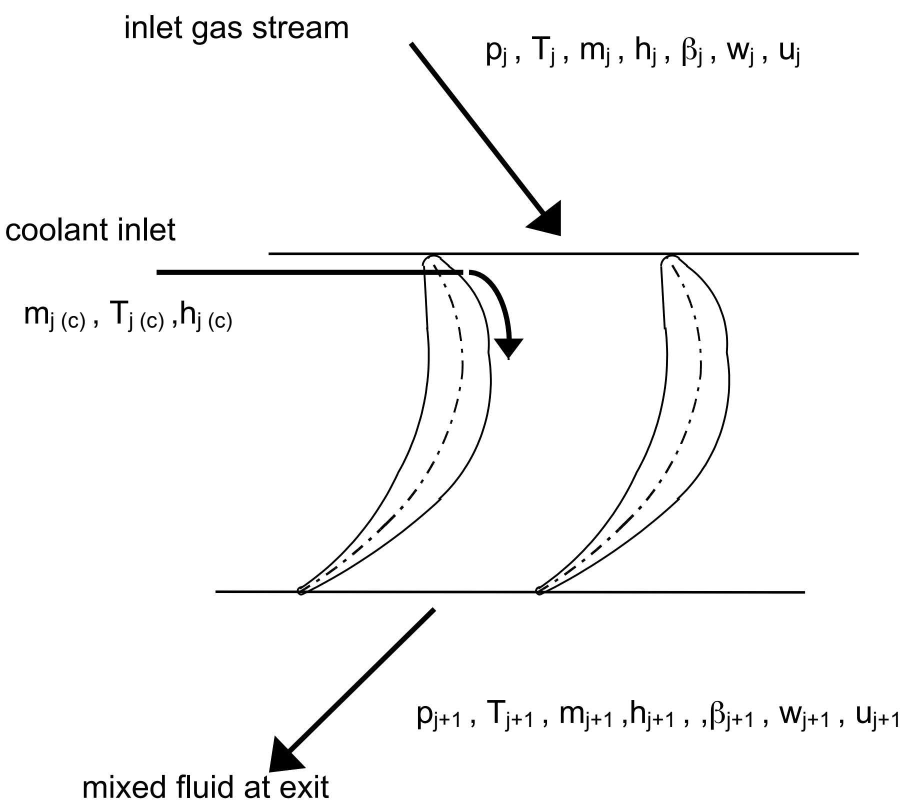


Fig. 4.17 Model of the cascade with film-cooling.

- <u>Main gas stream</u>

The equation of conservation of energy is expressed in term of total enthalpy of the rotor ($h°_{r(g)}$:):

$$h°_{r(g)} = h_j + \frac{1}{2} w^2_j - \frac{1}{2} u^2_j = h_{j+1(g)} + \frac{1}{2} w^2_{j+1(g)} - \frac{1}{2} u^2_{j+1} \tag{4.129}$$

where (g) denotes the condition of the gas expansion in the last stage before mixing with the coolant flow.

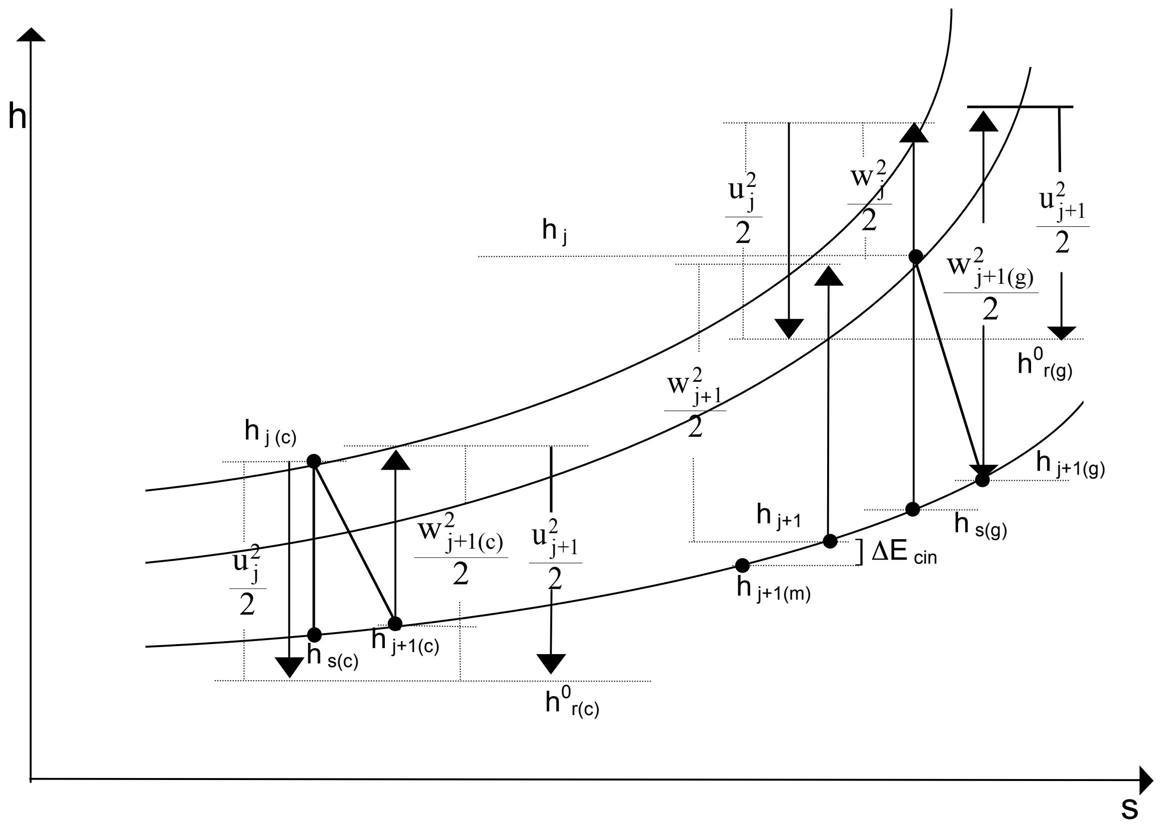


Fig. 4.18 Film cooling expansion h-S diagram

The velocity ($w_{j+1(g)}$) is determined by,

$$w_{j+1(g)} = \psi w_{s(g)} \tag{4.130}$$

(ψ) being the coefficient of the velocity loss, and ($w_{s(g)}$) is the velocity of the isentropic expansion given as:

$$w_{s(g)} = \sqrt{2(h_j - h_{s(g)}) + w_j^2 - u_j^2 + u_{j+1}^2} \tag{4. 131}$$

in this equation ($h_{s(g)}$ is calculated from the state equation of fluid as a function of compositions, pressure and temperature. Isentropic expansion temperature is calculated by:

$$T_{s(g)} = T_j \left(\frac{p_{j+1}}{p_j}\right)^{\frac{Rg}{c^*_p}} \tag{4. 132}$$

with (c^*_p) is the arithmetic mean of the specific heat at constant temperature (T_j) and (T_{j+1}). The enthalpy of the gas ($h_{j+1(g)}$) is given by:

$$h_{j+1(g)} = h^{\circ}_{r(g)} - \frac{1}{2} w^2_{j+1(g)} + \frac{1}{2} u^2_{j+1} \tag{4.133}$$

- Cooling flow

The coolant relative velocity is considered equal to zero i.e. $w_{j(c)} = 0$. The same approach for the main stream of gas is being adopted here, using the energy conservation equation:

$$h^{\circ}_{r(c)} = h_{j(c)} - \frac{1}{2} u^2_{j} = h_{j+1(c)} + \frac{1}{2} w^2_{j+1(c)} - \frac{1}{2} u^2_{j+1} \tag{4.134}$$

the relative velocity of the fluid is:

$$w_{j+1(c)} = \psi_c w_{s(c)} \tag{4.135}$$

the isentropic velocity at last point of expansion is given by:

$$w_{s(c)} = \sqrt{2(h_{j(c)} - h_{s(c)}) + w_j^2 - u_j^2 + u_{j+1}^2} \tag{4.135}$$

isentropic expansion temperature is:

$$T_{s(c)} = T_{j(c)} \left(\frac{p_{j+1}}{p_j} \right)^{\frac{Rc}{c_{pc}}} \tag{4.136}$$

enthalpy of cooling fluid at exit is:

$$h_{j+1(c)} = h^{\circ}_{r(c)} - \frac{1}{2} w^2_{j+1(c)} + \frac{1}{2} u^2_{j+1} \tag{4.137}$$

Mixing take place at the exit section of the blade at the pressure (pj+1.).
mass conservation

$$m_{j+1} = m_{j(c)} + m_j \tag{4.138}$$

The velocity of mixed fluid is obtained using the momentum equation:

$$m_{j+1} w_{j+1} = m_j\, w_{j+1(g)} + m_{j(c)} w_{j+1(c)} \tag{4.139}$$

Enthalpy of mixed fluid is given by:

$$h_{j+1} = \frac{m_j h_{j+1(g)} + m_{j(c)} h_{j+1(c)}}{m_{j+1}} + \Delta E_{cin} \tag{4.140}$$

in this equation (ΔE_{cin}) is the kinetic energy loss in the mixture:

$$\Delta E_{cin} = \frac{m_j w^2{}_{j+1(g)} + m_{j(c)} w^2{}_{j+1(c)} - m_{j+1} w^2{}_{j+1}}{2 m_{j+1}} \tag{4.141}$$

The mechanical power (P) exchanged between blade and fluid is calculated using the momentum equation:

$$P = m_j u_j\,(u_j - w_j \cos\beta_j) - m_{j+1} u_{j+1}(u_{j+1} - w_{j+1} \cos\beta_{j+1}) \tag{4.142}$$

($\cos\beta_j$) and ($\cos\beta_{j+1}$) being the fluid angles at inlet and exit, as shown in figure 4.17.

In the case of stator blade given in the above relations, the velocity of blade (u_j) and (u_{j+1}) are equal to zero. So the power in stator is zero, thus the power exchanged between fluid and blade using momentum equation is the one given in equation 4.142.
The phenomena of heat transfer between blade and fluid is calculated from the efficiency of coolant given by:

$$\varepsilon_c = \frac{T^{\circ}_{j(g)} - T_{b,j}}{T^{\circ}_{j(g)} - T_{j(c)}} \tag{4.143}$$

where($T^{\circ}_{j(g)}$) is the total temperature of the gas, ($T_{b,j}$) is the blade temperature and, and (T_c) is the cooling fluid temperature. The efficiency of the coolant is obtained from the empirical correlations generated from the curves.

4.3.7.1 Model for calculating the cycle

A similar thermodynamic differential equation model adopted for the axial compressor is used for the gas expander:

$$\frac{dT}{T} = \frac{k-1}{k} \cdot \eta_p \cdot \frac{dp}{p} \tag{4.144}$$

η_p being the polytropic efficiency. The calculation of the expansion line is same as the procedure adopted for the axial compressor.

The number of stages are established on the basis of cycle calculation by means of empirical correlations based on the manufacturer data.

4.3.7.2 Sizing

the following quantities that are necessary for successive off-design behaviour calculation are established:

- Hub diameter and blade height at each stage;
- Blade constructive angles;
- Velocity reducing coefficients in reference conditions (one for each row);
- Coolant flow rates and efficiencies in reference conditions;
- Pressure differences in reference conditions between coolant flow extraction point and the injection point in the principal gas flow.

For the calculation, the following quantities values are requested

- fluid inlet mass flow (sum del principal gas flow in the machine inlet section and coolant flows) determined in the previous cycle calculation;
- fraction of the gas compositions;
- fraction of the cooling fluid compositions
- temperature ISO;
- gas total inlet pressure at inlet;
- pressure in exit;
- number of stages;
- rotational speed;
- temperature of the blade;
- cooling fluid temperature.

The value of the mechanical efficiency is assumed equal to 0.999, this quantity is not accessible by the user; this choice is a result of adequacy. Such value could be modified including mechanical modules that take journal and trust bearings and machine weight into account. Moreover, the rates between each stage hub diameter and the diameter relative to the first row (stage n°1) are assigned.

The assumed hypothesis for the calculation are:

- axial direction of machine inlet flow;
- incidence angle equal to zero of the flow at each row inlet;
- Stage degree of reaction equal to 0.5;
- Blades designed in order to have a rectilinear suction face between the throat section and the outlet section.

The velocity triangles are established in order to obtain high values of blade efficiency. Such condition is achieved imposing a relationship between load coefficient Ψ (rate between the stage work and square blade velocity) and the flow coefficient ϕ (rate between the rotor inlet axial velocity and the blade velocity).
The curve of such a correlation is reported in fig 4.19.

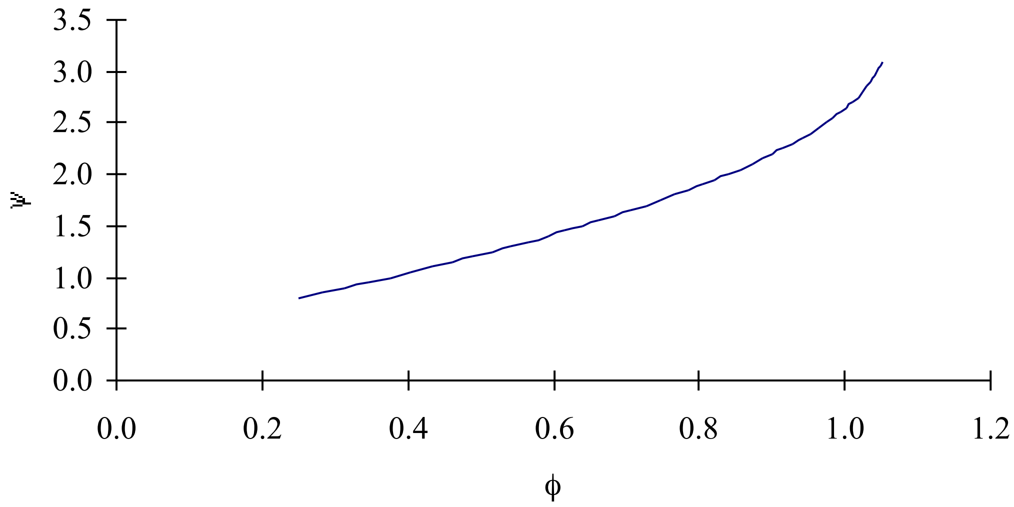


Fig. 4.19: Correlation ϕ-ψ

In order to completely define the velocity triangles, the rate between the rotational velocity and the rotor outlet one are assigned.

$$w_{j+1}cos\beta_{j+1}/\,u_{j+1} = 1.02 \qquad (4.145)$$

There are two further conditions concerning the total flow rate entering the machine and the ISO temperature.
The first expression is:

$$m= m_1 + \sum_{j=1}^{j=Nsr} m_{j(c)} \qquad (4.146)$$

m_1 being the flow at inlet to the machine (station 1) and ($m_{j(c)}$) being the coolant flow relative to the number of stators and rotors being cooled.

The second expression imposed on the value of the inlet temperature at ISO condition is:

$$mh(T_{iso}) = m_1h(T_1) + \sum_{j=1}^{j=Nsr} m_{j(c)}\, h_c(T_c) \qquad (4.147)$$

$h(T_{iso})$, $h(T_1)$ and $h_c(T_c)$ being the enthalpies at ISO temperature, gas at inlet to the first stage and coolant temperature, respectively.
The coolant flow are determined by mean of the relationships deducted from available data of existing machinery.
Such expressions provide the fraction values:

$$\mu_j = (m_{j(c)}/m_j\,)\ 100 \qquad (4.148)$$

In function of the difference between the row inlet gas temperature and the blade temperature at reference conditions $T^*_{j(b)}$:

$$\mu_j = A + B(\,T°_j - T^*_{j(b)}) \qquad (4.149)$$

The value of the coefficients A and B assumed for stator blades (used for film-cooling) are:

$A = 0.0845$ $\qquad$ $B= 0.0105$

The value of the coefficients A and B assumed for rotor blades (used for film-cooling) are:

$A = 0.139$ $B= 0.0284$

The coefficients predicted for the velocities of ($\psi_{s(ref)}$) and ($\psi_{r(ref)}$) related to stator and rotor blades introduced in eq.4 are calculated using the correlation reported in the literature given in figure 4.20 and 4.21. ($\psi_{s(ref)}$) is expressed as function of the fluid angle at stator exit condition, and ($\psi_{r(ref)}$) is obtained as a function o deflection i.e. $\Delta\beta = \beta_{j+1}-\beta_j$. For the coefficient predicting the velocity of the coolant (ψ_c) a constant value equal to 0.6 is assumed.

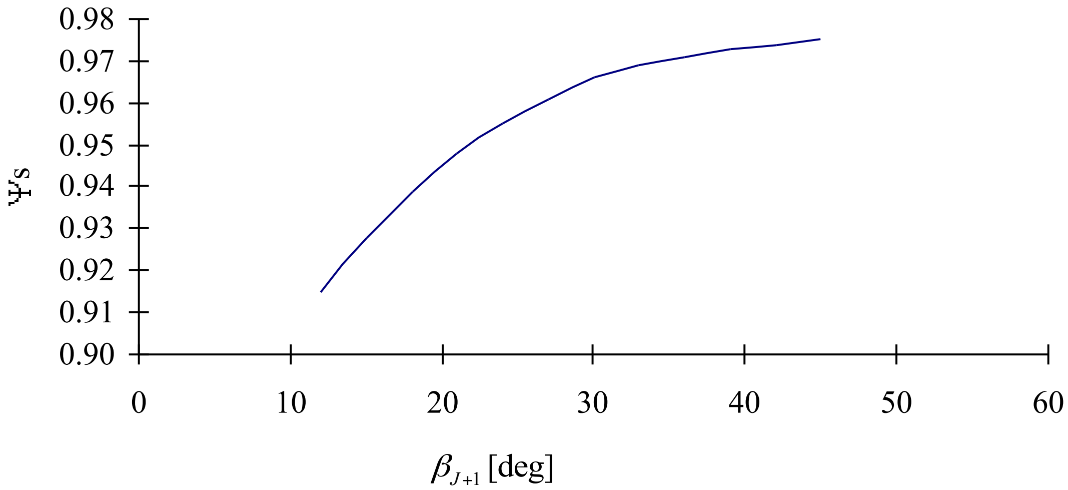


Fig. 4.20 Coefficient for predicting the velocity of stator blade.

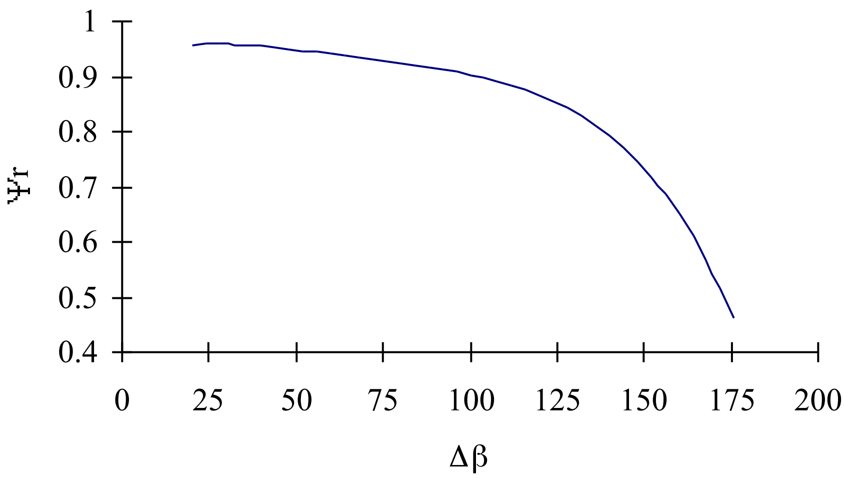


Fig. 4.21 Coefficient predicting the velocity of rotor blade.

Blade angle at the leading edge is determined by zero-incidence flow condition. At trailing edge the flow angle (β_{j+1}) and blade angle ($\beta_{j+1(d)}$) (expressed in radiant) are calculated as a function of the local Mach number (M_{j+1}):

- For $M_{j+1} \leq 0.5$ $\beta_{(0.5)j+1} = 1.12\beta_{(d)j+1} - 0.0279$ (4.150)

- For $M_{j+1} = 1$ $\beta_{(1.0)j+1} = \beta_{(d)j+1}$ (4.151)

For the values of Mach number between 0.5 and 1 external values are connected by a sinusoid . The relation is built up by sinusoid line according the extreme values.

The mechanical power and variables whose values do not change during calculation has already been discussed in axial flow compressors.
The equation of energy conservation is written as:

$$m\,(h_u-h_i) = P_{mu}-P_{mi}+P^*_{mp} \tag{4.152}$$

where (h_i) and (h_u) respectively are the enthalpies at inlet and exit, P^*_{mp} is the mechanical power lost.
(m) represents the overall gas mass flow entering the gas expander, i.e. the main stream gas flow at combustion chamber exit and the air flow requested for the blade cooling. The temperature (T_i) is defined according to ISO condition: T_i represents the temperature obtained by mixing all the mass flows entering the gas expander.

Mechanical power losses (P^*_{mp}) are given by:

$$P^*_{mp} = \frac{1-\eta_m}{\eta_m} P_m \tag{4.153}$$

with $P_m = P_{mu} - P_{mi}$ being the mechanical power output. The constant (K_e) is employed to calculate mechanical losses occurring during off-design options:

$$K_e = P^*_{mp}/n^* \tag{4.154}$$

the term (n*) indicate the value of rotational speed at nominal condition.

4.3.7.3 Part Load Analysis

Figure4.22 shows the schematic diagram of a gas expander:

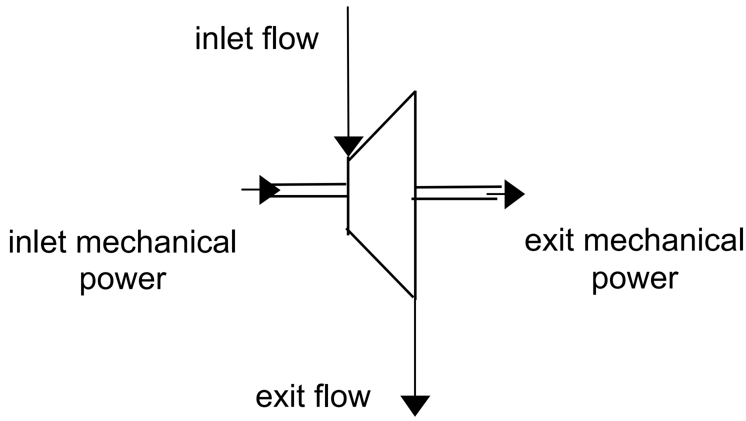


Fig. 4.22: Schematic model of a gas expander.

The inlet and exit variable quantities in each junctions are:

inlet flow

m	mass flow	[kg/s]
p_i	pressure	[MPa]
T_{iso}	temperature (ISO)	[°C]

composition (fraction in mass O_2, N_2, CO_2, H_2O)

exit flow

m	mass flow	[kg/s]
p_u	pressure	[MPa]
T_u	temperature	[°C]

composition (fraction in mass O_2, N_2, CO_2, H_2O)

inlet mechanical power

P_{mi}	mechanical power	[MW]
n	rotational velocity	[rpm]

exit mechanical power

P_{mu}	mechanical power	[MW]
n	rotational velocity	[rpm]

The gas expander model has been developed to analyze the off-design behaviour both for cooled and uncooled. A mean line row by row calculation method has been implemented. Input geometry data obtained by inverse sizing are given as a input data.

The input geometry data for gas-expander with cooling are:

- stator inlet area [m^2];
- stator inlet blade angle [°];
- stator exit area = rotor inlet area [m^2];
- stator mean diameter [m];
- stator exit blade angle [°];
- stator efficiency and correction coefficient;

- rotor inlet blade angle [°];
- rotor exit area [m^2];
- rotor mean diameter [m];
- rotor blade exit angle [°]
- rotor efficiency and correction coefficient;
- maximum permissible temperature of stator bade [°C];
- maximum permissible temperature of rotor blade [°C];
- cooling temperature for the stator blade [°C];
- cooling temperature for the rotor blade [°C];

The corresponding inlet and exit variables across gas expander with cooling are:

- stage inlet mass flow [kg/s];
- stage inlet pressure [kPa];
- inlet temperature [°C];
- fluid inlet angle [°];
- inlet fraction of mass compositions (consisting of: O_2, N_2, CO_2, H_2O) [%]
- stator exit pressure [kPa];
- stator exit temperature [K];
- stator fluid exit angle [°];
- rotor inlet pressure [kPa];
- rotor inlet temperature [K];
- rotor fluid inlet angle [°];
- stage exit pressure [kPa];
- stage exit temperature [K];
- stage fluid exit angle [°];
- stage exit mass flow [kg/s];
- exit fraction of mass compositions (consisting of: O_2, N_2, CO_2, H_2O); [%]
- stator mass flow of cooling fluid [kg/s];
- stator cooling fluid temperature [K];
- rotor mass flow of cooling fluid [kg/s];
- rotor cooling fluid temperature [K];
- fractions of mass compositions of cooling fluid (consisting of: O_2, N_2, CO_2, H_2O); [%]
- rotational speed.

The different operating points are determined giving the values of:

- gas mass flow at inlet [kg/s];
- expander inlet pressure [kPa], and temperature[°C];
- rotational speed; and
- exit pressure [kPa].

Blade cooling has been taken into account using a set of cooling effectiveness versus coolant mass flow curves given in non-dimensional form.

To solve direct problem geometrical quantities are assigned as data. Conditions on overall mass flow entering the machine and on ISO temperature.
The first expression is:

$$m = m_1 + \sum_{j=1}^{j=Nsr} m_{j(c)} \tag{4.155}$$

$m_1$being the flow at inlet to the machine (station 1) and ($m_{j(c)}$) being the coolant flow relative to the number of stators and rotors being cooled, respectively.

The second condition imposed on the value of the inlet temperature at ISO condition is:

$$mh(T_{iso}) = m_1 h(T_1) + \sum_{j=1}^{j=Nsr} \mathrm{m}_{j(c)}\, h_c(T_c) \tag{4.156}$$

$h(T_{iso})$, $h(T_1)$ and $h_c(T_c)$ being the enthalpies at ISO temperature, gas at inlet to the first stage and coolant temperature, respectively

In solving direct problem the above quantity are determined according to matching conditions with other components consisting the plant. As previously stated at reference condition the flow is sub-sonic. However in off-design operations could lead to sonic flow in one or more blade rows. To establish if sonic speed is reached the properties of main stream gas at row inlet are taken into consideration. The fluid is taken as perfect gas, thus critical temperature is given by:

$$T_{cr} = \left(\frac{u_j^2 - u_{j+1}^2}{2c_p} + T_j^0 \right) \frac{2}{k+1} \tag{4.157}$$

being

$$T^0_j = T_j + w^2_j / c_p \tag{4.158}$$

is the total relative temperature to the row, (c_p) and (k) are the specific heat at constant pressure and specific heat ratio evaluated as a function of (T_j):

Sonic velocity is given by:

$$c_s = \sqrt{k^* R_g T_{cr}} \tag{4.159}$$

(k^*) being the mean specific ratio between(T_j) and (T_{j+1}).

The critical pressure depends on the losses which occur during the expansion. The speed loss coefficient for critical flow (ψ_{cr}) is evaluated using eq. 33 (introduce in following) where the value of Mach number is equal to one (Ma=1). (p_{cr}) is given by:

$$p_{cr} = p°_j \left[\left(1 - \frac{u_{j+1}^2 - u_j^2}{2c_p} \right) \left(1 - \frac{k^* - 1}{k^* + 1} \cdot \frac{1}{\psi_{cr}{}^2} \right) \right]^{\frac{c_p}{R_g}} \tag{4.160}$$

If a stationary blade row is taken into consideration in eq. 4.160 blade velocities are put equal to zero.

Critical mass flow is expressed by:

$$m_{cr} = \frac{p_{cr}}{RT_{cr}} A_{j+1} c_s senb_{j+1} \tag{4.161}$$

(A_{j+1}) representing the flow area at the exit of the row.

If critical flows occur (i.e. the values found for critical pressure and temperature are greater than p_{j+1} and T_{j+1} at row exit the following conditions are imposed:

$$m_{j+1} = m_{cr} \tag{4.162}$$

The presence of air cooling flow extractions is supposed to be equal to the gas expander inlet pressure. This pressure level (not considering the pressure loss occurring in combustion chamber) represents the discharge pressure of the compressor moved by the gas expander. Coolant mass flows are related to pressure drop between gas expander inlet and blade row exit by means of the following expression:

$$\frac{p_1^0 - p_{j+1}}{p_1^0} = K_{p,j}\left(\frac{m_{(c)j}}{p_1^0}\right)^2 \tag{4.163}$$

($K_{p,j}$) being established on the basis of reference quantities value:

$$K_{p,j} = \left(\frac{p_1^0 - p_{j+1}}{p_1^0}\right)_{ref}\left(\frac{p_1^0}{m_{(c)j}}\right)^2_{ref} \tag{4.164}$$

The relationship between blade cooling effectiveness and the ratio $\mu = (m_{(c)}/m_j)$ is expressed by an empirical generalized curve reported in figure 4.23 (μ) and (ε) values are normalized with respect to reference value.

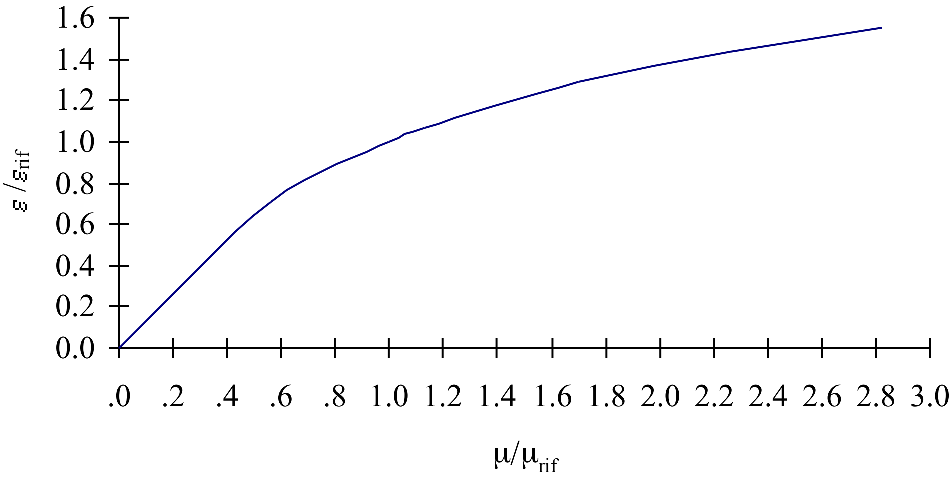


Fig. 4.23: generalized effectiveness vs corrected mass flow curves

Blade temperature is evaluated taking eq. 4.143 into account:

$$T_b = T_g - \varepsilon \cdot (T_g - T_c) \tag{4.143}$$

Main gas stream velocity loss coefficient is evaluated by modifying the reference velocity loss coefficient ($\psi_{(ref)}$) as a function of the incidence angle and the Mach number at the exit of the row.
For a sub sonic flow (ψ) is calculated as follows:

$$\psi^2 = \psi^2_{(ref)} - CKT\tan^2 i \tag{4.165}$$

(CKT) being a coefficient which takes into account blade profile sensitivity to changes of incidence angle. A value of CKT= 0.05 has been assumed for stator vanes. For rotor blades CKT = 0.05.
If the flow is supersonic the velocity loss coefficient is calculated by:

$$\psi^2 = 1 - (1 - \psi^2_{(ref)} + CKT\tan^2 i \left[1 + 60(1 - \mathrm{Ma})^2 \frac{2 + k}{2k + k\mathrm{Ma}^2}\right] \tag{4.166}$$

(k) taken as mean value between row inlet and outlet.
For coolant expansion a constant value of $\psi_{(c)} = 0.6$ is being considered.
If the flow is sub-sonic the blade angle and flow angle at the blade trailing edge is calculated by eq. 4.158 and 4.162. If the flow is supersonic the exit flow angle is calculated applying the continuity equation between throat section and exit section j+1:

$$\rho_{cr} c_s \sin\beta_{j+1(d)} = \rho_{j+1} w_{j+1} \sin\beta_{j+1} \tag{4.167}$$

$\sin\beta_{j+1(d)}$ being the flow angle at throat section, taken equal to the blade angle according to eq. 4.158

4.3.8 Surface Heat Exchangers

Heat transfer by convection is the phenomenon that prevails in units of heat recovery systems placed downstream of gas expanders. In this zone there are burnt gas/steam (super-heaters), burnt gas/saturated steam (evaporators) and burnt gas/water exchangers. Units of steam generators placed downstream combustion chamber, condensers and feed-water regenerators are of connective type too.

Suitable equipment's to take advantage of this kind of heat transfer phenomenon are heterogeneous systems constituted by three spatially distinct subsystems: cold fluid, tube walls and hot fluid, interacting through boundary surface.

The model is based on ε-NTU approach (effectiveness-Number of Transfer Units), mass and energy conservation and constitutive equations.

<u>Heat Recovery Systems</u>

Nomenclature:

- m = mass flow
- T = temperature
- p = pressure
- h = enthalpy
- P = mechanical power
- Q = thermal power
- * = reference value

Data for heat recovery systems

Input data are:

<u>Evaporator</u>

- minimum difference between saturation temperature and inlet temperature ΔTmin [°C]
- heat transfer area [m^2];
- design outlet steam temperature [°C];
- design steam mass flow [kg/s];
- heat transfer coefficient of steam [W/m^2];
- heat transfer coefficient of gas [W/m^2];
- gas mass flow [kg/s];
- gas mass compositions [%].

<u>Superheater</u>

- heat transfer area [m^2]
- design inlet steam temperature [°C]
- design outlet steam temperature [°C]
- design steam mass flow [kg/s]
- heat transfer coefficient of steam [W/m^2]
- heat transfer coefficient of gas [W/m^2]
- gas mass flow [kg/s]
- gas mass compositions [%]

<u>Economiser</u>

- heat transfer area [m^2]
- design outlet water temperature [°C]
- design inlet water temperature [°C]
- design water mass flow [kg/s]
- heat transfer coefficient of water [W/m^2]
- heat transfer coefficient of gas [W/m^2]
- gas mass flow [kg/s]

- gas mass composition [%]

These data are used to obtain the reference values in design condition. The obtained values are stored in a input data file and are used by COMD code for the off design analyses.

4.3.9 Evaporator of waste heat boiler

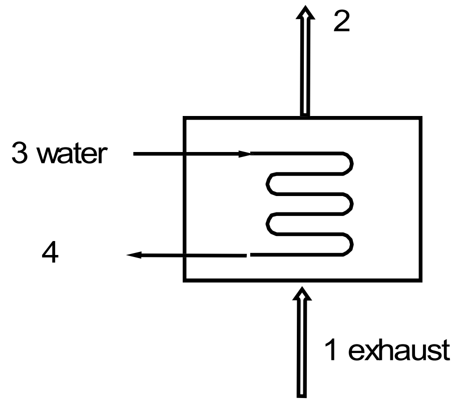


Fig. 4.24: generic evaporator scheme

Evaporator is described schematically in figure 4.24. Water coming from drum enter into the unit and receive heat by exhaust gas. Input data are those established by design condition and the parameters (factor ff_b taking into account the fouling, and gas pressure loss kp_{gb}) describing the status of the component.

The governing equations are:

mass conservation:

$$m_1 - m_2 = 0 \tag{4.168}$$

$$m_3 - m_4 = 0 \tag{4.169}$$

energy balance:

$$(m_1 \cdot h_1 - m_2 \cdot h_2) - (m_4 \cdot h_4 - m_3 \cdot h_3) = 0 \tag{4.170}$$

heat transfer:

$$Q_b - (m_1 \cdot h_1 - m_2 \cdot h_2) = 0 \tag{4.171}$$

$$Q_b - \varepsilon \cdot c_{\min} \cdot (T_1 - T_3) = 0 \tag{4.172}$$

$c_{\min}$ being the hot fluid (exhaust) specific heat.

$$\varepsilon - (1 - e^{-NTU}) = 0 \tag{4.173}$$

$$NTU - f_1(NTU^*, FFb, m_1, m_3, T_1, T_2, T_3, T_4, [xx]_{GAS}) \quad (4.174)$$

The above equation is solved using the following relationships:

steam heat transfer coefficient:

$$U_{sb} = U_{sb}{}^* \cdot \left(\frac{m_4}{m_4^*}\right)^{0.1666} \quad (4.175)$$

exhaust heat transfer coefficient:

$$U_{gb} = U_{gb}{}^* \cdot \left(\frac{m_1}{m_1^*}\right)^w \cdot \left(\frac{\mu_g}{\mu_g{}^*}\right)^x \cdot \left(\frac{\lambda_g}{\lambda_g{}^*}\right)^y \cdot \left(\frac{c_{pg}}{c_{pg}^*}\right)^z \quad (4.176)$$

μ_g, λ_g, c_{pg} being the dynamic viscosity [Pa-s], thermal conductivity [kW/ (m K)] and the heat capacity [kJ/(kg s)] of the gas. The exponent coefficients *w, x, y, z* assume different value in dependence on the fluid of interest. Table below shows the value of these coefficients.

	gas	Water/steam
w	0.6	0.8
x	-0.27	-0.47
y	0.67	0.67
z	0.33	0.33

overall heat transfer coefficient:

$$U_{bc} = \frac{1}{\frac{1}{U_{sb}} + \frac{1}{U_{gb}}} \quad (4.177)$$

$$U_b = U_{bc} \cdot ff_b \quad (4.178)$$

$$NTU = NTU^* \cdot \frac{c_{\min}}{c_{\min}^*} \cdot \frac{U_b}{U_b^*} = \frac{U_b \cdot S}{c_{\min}} \quad (4.179)$$

S being the heat transfer surface.

Other equations used are:

exhaust pressure drop:

$$\Delta p_{gb} - \frac{kp_{gb} \cdot m_1^2 \cdot T_1}{p_1} = 0 \quad (4.180)$$

fluid properties:

$h_1 - H(p_1, T_1, [xx]_{gas}) = 0$ (4.181)

$h_2 - H(p_2, T_2, [xx]_{gas}) = 0$ (4.182)

$h_3 - H(p_3, T_3) = 0$ (4.183)

$p_4 - p(T_4) = 0 \rightarrow equilibrium$ (4.184)

The following inequality constrains are imposed:

$$T_4 - T_3 > 0$$
$$T_1 - T_4 > 0$$
$$T_2 - T_3 > 0$$

$$pinch\ point > 0$$

$$T_4 - (T_1 + \Delta T_{\min}) > 0$$

4.3.10 Super-heater of waste heat boiler

Super-heater is described schematically in figure 4.25. Steam coming from drum enter into the unit and receive heat by exhaust gas. Input data are those established by design condition and the parameters (factor ff_s taking into account the fouling, and pressure loss for exhaust kp_{gs} and steam kp_{ss}) describing the status of the component.

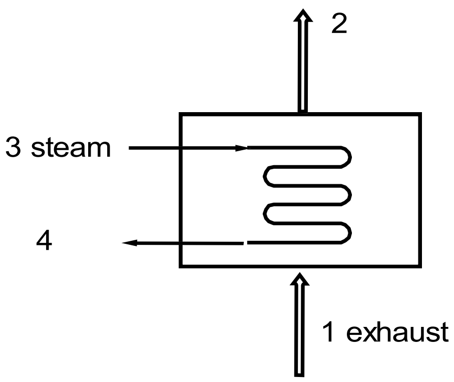


Fig. 4.25: generic super-heater scheme

The governing equations are:

mass conservation:

$m_1 - m_2 = 0$ (4.185)

$$m_3 - m_4 = 0 \quad (4.186)$$

energy balance:

$$(m_1 \cdot h_1 - m_2 \cdot h_2) - (m_4 \cdot h_4 - m_3 \cdot h_3) = 0 \quad (4.187)$$

heat transfer:

$$Q_b - (m_1 \cdot h_1 - m_2 \cdot h_2) = 0 \quad (4.188)$$

$$Q_b - \varepsilon \cdot c_{\min} \cdot (T_1 - T_3) = 0 \quad (4.189)$$

$$\varepsilon - \frac{1 - e^{-NTU\cdot(1-\kappa)}}{1 - \kappa \cdot e^{-NTU\cdot(1-\kappa)}} = 0 \quad (4.190)$$

$$\kappa = \frac{c_{\min}}{c_{\max}} \quad (4.191)$$

For software reasons a control is established on (κ), if it is less than 0.0001 the above equation is changed as following:

ε - [1-exp(-NTU)]= 0 (4.192)

and if (1-κ) is less than 0.0001

ε - NTU/(1+NTU) = 0 (4.193)

steam heat transfer coefficient:

$$U_{ss} = U_{ss}^{\ *} \cdot \left(\frac{m_3}{m_3^*}\right)^w \cdot \left(\frac{\mu_s}{\mu_s^*}\right)^x \cdot \left(\frac{\lambda_s}{\lambda_s^*}\right)^y \cdot \left(\frac{c_{ps}}{c_{ps}^*}\right)^z \quad (4.194)$$

μ_s, λ_s, c_{ps} being the dynamic viscosity [Pa-s], thermal conductivity [kW/ (m K)] and the heat capacity [kJ/(kg s)] of the steam. The exponent coefficients *w, x, y, z* assume different value in dependence on the fluid of interest. Table below shows the value of these coefficients.

exhaust heat transfer coefficient:

$$U_{gs} = U_{gs}^{\ *} \cdot \left(\frac{m_1}{m_1^*}\right)^w \cdot \left(\frac{\mu_g}{\mu_g^*}\right)^x \cdot \left(\frac{\lambda_g}{\lambda_g^*}\right)^y \cdot \left(\frac{c_{pg}}{c_{pg}^*}\right)^z \quad (4.195)$$

μ, λ, c_p being the dynamic viscosity [Pa-s], thermal conductivity [kW/ (m K)] and the heat capacity [kJ/(kg s)] of the gas. The exponent coefficients *w, x, y, z* assume different value in dependence on the fluid of interest. Table below shows the value of these coefficients.

	gas	Water/steam
w	0.6	0.8
x	-0.27	-0.47
y	0.67	0.67
z	0.33	0.33

overall heat transfer coefficient:

$$U_{sc} = \frac{1}{\dfrac{1}{U_{ss}} + \dfrac{1}{U_{gs}}} \tag{4.196}$$

$$U_s = U_{sc} \cdot ff_s \tag{4.197}$$

$$NTU = \frac{U_s \cdot S_s}{c_{\min}} \tag{4.198}$$

S_s being the heat transfer surface.

Other equations used are:

exhaust pressure drop:

$$\Delta p_{ss} - kp_{ss} \cdot m_3^2 \cdot \rho_s = 0 \tag{4.199}$$

$\rho_s\ [kg \cdot m^{-3}]$ being the steam density

exhaust pressure drop:

$$\Delta p_{gs} - \frac{kp_{gs} \cdot m_1^2 \cdot T_1}{p_1} = 0 \tag{4.200}$$

fluid properties:

$$h_1 - H(p_1, T_1, [xx]_{gas}) = 0 \tag{4.201}$$

$$h_2 - H(p_2, T_2, [xx]_{gas}) = 0 \tag{4.202}$$

$$h_3 - H(p_3, T_3) = 0 \tag{4.203}$$

$$h_4 - H(p_4, T_4) = 0 \tag{4.204}$$

The following inequality constrains are imposed:

$$T_4 - T_3 > 0$$

$T_1 - T_4 > 0$
$T_2 - T_3 > 0$
$T_4 - T_3 > 0$

4.3.11 Economiser for heat recovery by warm water

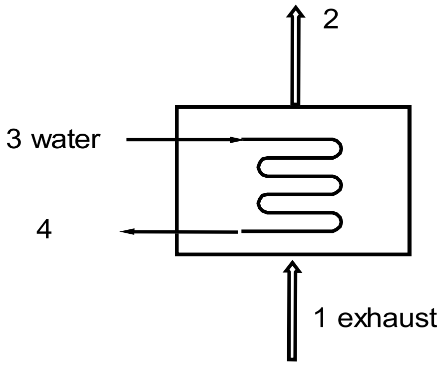


Fig. 4.26: general economizer scheme

Economiser is described schematically in figure 4.26. Water enter into the unit and receive heat by exhaust gas coming from waste heat boiler. Input data are those established by design condition and the parameters (factor ff_e taking into account the fouling, and pressure loss for exhaust kp_{ge} and steam kp_{se}) describing the status of the component.
The governing equations are:

mass conservation:

$$m_1 - m_2 = 0 \tag{4.205}$$

$$m_3 - m_4 = 0 \tag{4.206}$$

energy balance:

$$(m_1 \cdot h_1 - m_2 \cdot h_2) - (m_4 \cdot h_4 - m_3 \cdot h_3) = 0 \tag{4.207}$$

heat transfer:

$$Q_b - (m_1 \cdot h_1 - m_2 \cdot h_2) = 0 \tag{4.208}$$

$$Q_b - \varepsilon \cdot c_{\min} \cdot (T_1 - T_3) = 0 \tag{4.209}$$

$$\varepsilon - \frac{1 - e^{-NTU \cdot (1-\kappa)}}{1 - \kappa \cdot e^{-NTU \cdot (1-\kappa)}} = 0 \tag{4.210}$$

$$\kappa = \frac{c_{\min}}{c_{\max}} \tag{4.211}$$

For software reasons a control is established on (κ), if it is less than 0.0001 the above equation is changed as following:

$$\varepsilon - [1-\exp(-NTU)] = 0 \tag{4.212}$$

and if (1-κ) is less then 0.0001

$$\varepsilon - NTU/(1+NTU) = 0 \tag{4.213}$$

<u>steam heat transfer coefficient</u>:

$$U_{se} = U_{se}{}^{*} \cdot \left(\frac{m_3}{m_3^*}\right)^w \cdot \left(\frac{\mu_s}{\mu_s{}^*}\right)^x \cdot \left(\frac{\lambda_s}{\lambda_s{}^*}\right)^y \cdot \left(\frac{c_{ps}}{c_{ps}^*}\right)^z \tag{4.214}$$

μ_s, λ_s, c_{ps} being the dynamic viscosity [Pa-s], thermal conductivity [kW/ (m K)] and the heat capacity [kJ/(kg s)] of the steam. The exponent coefficients *w, x, y, z* assume different value in dependence on the fluid of interest. Table below shows the value of these coefficients.

<u>exhaust heat transfer coefficient</u>:

$$U_{ge} = U_{ge}{}^{*} \cdot \left(\frac{m_1}{m_1^*}\right)^w \cdot \left(\frac{\mu_g}{\mu_g{}^*}\right)^x \cdot \left(\frac{\lambda_g}{\lambda_g{}^*}\right)^y \cdot \left(\frac{c_{pg}}{c_{pg}^*}\right)^z \tag{4.215}$$

μ, λ, c_p being the dynamic viscosity [Pa-s], thermal conductivity [kW/ (m K)] and the heat capacity [kJ/(kg s)] of the gas. The exponent coefficients *w, x, y, z* assume different value in dependence on the fluid of interest. Table below shows the value of these coefficients.

	gas	Water/steam
w	0.6	0.8
x	-0.27	-0.47
y	0.67	0.67
z	0.33	0.33

<u>overall heat transfer coefficient:</u>

$$U_{ec} = \frac{1}{\frac{1}{U_{se}} + \frac{1}{U_{ge}}} \tag{4.216}$$

$$U_s = U_{ec} \cdot ff_e \tag{4.217}$$

$$NTU = \frac{U_e \cdot S_e}{c_{\min}} \tag{4.218}$$

Se being the heat transfer surface.

Other equations used are:
exhaust pressure drop:

$$\Delta p_{se} - kp_{se} \cdot m_3^2 \cdot \rho_s = 0 \tag{4.219}$$

$\rho_s \, [kg \cdot m^{-3}]$ being the steam density

exhaust pressure drop:

$$\Delta p_{ge} - \frac{kp_{ge} \cdot m_1^2 \cdot T_1}{p_1} = 0 \tag{4.220}$$

fluid properties:

$$h_1 - H(p_1, T_1, [xx]_{gas}) = 0 \tag{4.221}$$

$$h_2 - H(p_2, T_2, [xx]_{gas}) = 0 \tag{4.222}$$

$$h_3 - H(p_3, T_3) = 0 \tag{4.223}$$

$$h_4 - H(p_4, T_4) = 0 \tag{4.224}$$

The following inequality constrains are imposed:

$$T_4 - T_3 > 0$$
$$T_1 - T_4 > 0$$
$$T_2 - T_3 > 0$$
$$T_4 - T_3 > 0$$

4.3.11 Steam Turbine

The model refers to a group of stages of a steam turbine. At inlet and outlet stations of the stage, admission and extraction of fluid has been taken into consideration as the following figure points out:

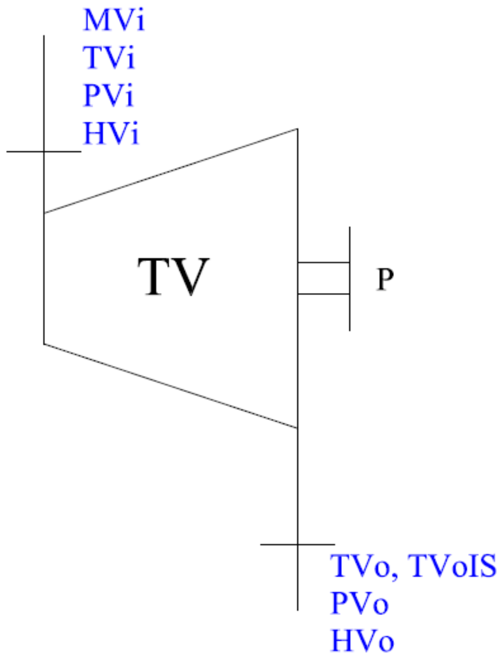


Fig. 4.27: Scheme of a generic steam expander

Taken the inlet and outlet section into account, some variables have been defined:

Inlet Section

$\dot{m}_i\,[kg \cdot s^{-1}]$	Inlet steam mass flow
$T_i\,[°C]$	Inlet steam temperature
$p_i\,[kPa]$	Inlet steam pressure
$h_i\,[kJ \cdot kg^{-1}]$	Inlet steam enthalpy
$n\,[rpm]$	Inlet shaft rotational speed
$P_{mi}\,[MW]$	Inlet mechanical power

Outlet Section

$\dot{m}_i\,[kg \cdot s^{-1}]$	Outlet steam mass flow
$T_o\,[°C]$	Outlet steam temperature
$p_o\,[kPa]$	Outlet steam pressure
$h_o\,[kJ \cdot kg^{-1}]$	Outlet steam enthalpy
$P_{mo}\,[MW]$	Outlet mechanical power

The model considers the mass flow constant through the stage. Therefore the conservation low of mass is satisfied as the following relation indicates:

$$\dot{m}_i = \dot{m}_o = \dot{m} \tag{4.225}$$

The conservation of energy is expressed as follows:

$$m \cdot (h_i - h_o) - P_{mL} = P_{mo} - P_{mi} \tag{4.226}$$

P_{mL} being the mechanical power losses.

At the inlet and outlet section the constitutive equations of the fluid have to be satisfied:

$$f(h, p, T) = 0 \tag{4.227}$$

4.3.11.1 model for the cycle calculation

Taken the cycle calculation into consideration, the model has been structured as well as the expander one. In this case a correction of the value of the polytropic efficiency has been carried out in dependence of the steam quality (the line expansion enters partially or fully in the saturated steam region). The polytropic efficiency η_p is expressed by:

$$\eta_p = \eta_{pd} - (1 - x) \tag{4.228}$$

η_{pd} being the polytropic efficiency of a steam expansion in the superheated region and x being the steam quality. The calculation has been carried out splitting up the pressure drop across the expander in a suitable number of intervals. For each interval equation 4.228 has been applied setting x to the initial condition of the expansion.

4.3.11.2 model to evaluate the reference quantities

The model takes empirical correlation gathered in literature into account and allows to evaluate the efficiency of a group of steam turbine stages in the referenced conditions. These conditions have been established by the cycle calculation or directly by the user:

- pressure, temperature or quality of the inlet steam
- outlet pressure
- steam mass flow
- rotationally speed

Correlations adopted are different in relation to the inlet – outlet state of the steam. Taking the end of the isentropic expansion into account, the suitable correlation can be chosen. In any case the procedure requires the calculation of a basic value of the efficiency that is obtained by suitable coefficients calculating according to eq. 4.232

$$\eta = \eta_b \cdot c_1 \cdot c_2 \tag{4.232}$$

Different cases have been taken into consideration:

1. Group of not regulated stages: total admission stages have been considered. Taken the admission or the extraction of steam into account, dissipative phenomena related to the outlet section have been considered assigning a pressure loss of 3% at the exhaust.

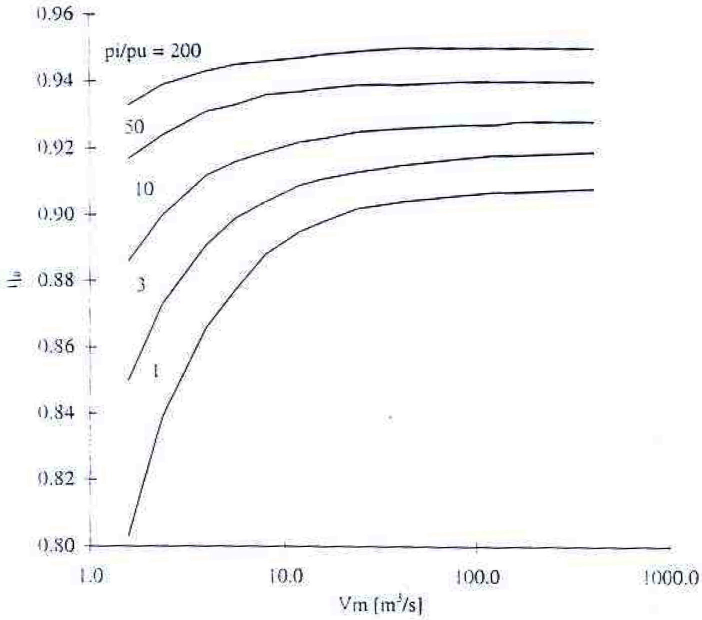


Fig. 4.28: efficiency of not regulated stages: superheated expansion

Three subcases are also taken into account

a) Expansion in the superheated region: the efficiency of the group has been determined taking the volumetric flow $V_m\ [m^3 \cdot s^{-1}]$ and the pressure ratio $\beta = \frac{p_i}{p_o}$ into account, as the figure 4.29 shows.
b) Expansion in the saturated region: the efficiency base η_b is evaluated as a function of the inlet pressure p_i and of the pressure ratio β, just defined. Efficiency is represented in figure 4.29.

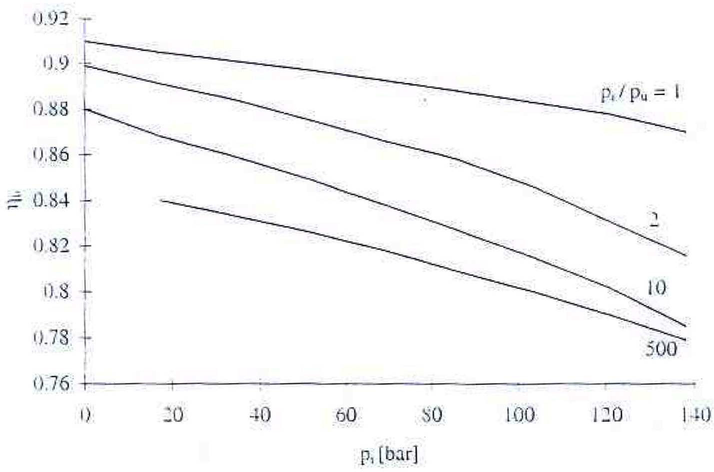


Fig. 4.29: efficiency of not regulated stages: saturated expansion

Corrections have been implemented taking the volumetric flow and the steam quality into account. The first coefficient c_1 is expressed as follows:

$$c_1 = \frac{f_1(\beta, V_m)}{f_1^*(\beta, V_m^*)} \tag{4.229}$$

f_1, f_1^* being the values plotted in figure 4.30 and related to the inlet volumetric flow V_m, V_m^*. V_m^* is moreover defined as follows:

$$V_m^* = 23.6 \cdot v^* \tag{4.230}$$

$v^*\,[m^3 \cdot kg^{-1}]$ being the specific volume of the superheated steam at the pressure inlet condition.

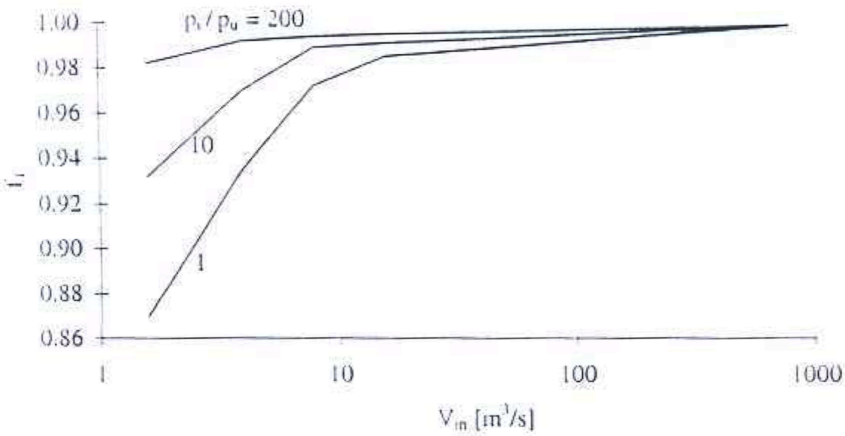


Fig. 4.30: Group of not regulated stages: saturated expansion. Corrective coefficient vs inlet volumetric flow

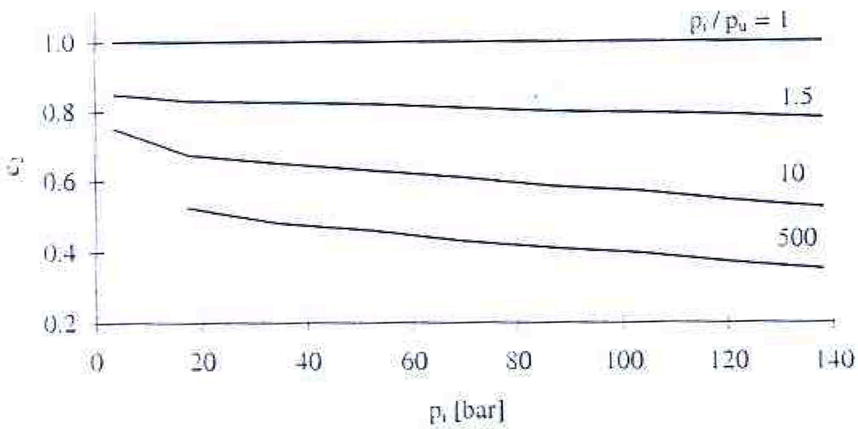


Fig. 4.31: Group of not regulated stages: saturated expansion. Corrective coefficient vs inlet steam quality

The second coefficient c_2 takes the inlet steam humidity into account:

$$c_2 = 1 - f_2(p_i, \beta) \cdot (1 - x_i) \tag{4.231}$$

x_i being the inlet steam quality and $f_2(p_i, \beta)$ being obtained by the curves plotted in figure 4.31. The efficiency is expressed by the following relation:

$$\eta = \eta_b \cdot c_1 \cdot c_2 \tag{4.232}$$

c) Expansion in both regions (superheated and saturated): if the isentropic line expansion begins in the superheated region and ends in the saturated one, the crossing pressure p_s of the dry saturated steam line has been determined. Efficiency in this circumstance has been evaluated taking two different contributions into consideration. The first one considers the expansion from the inlet pressure p_i to the p_s, in the superheated region and the efficiency is established with the previous correlations. The second one refers to the expansion in the saturated region that begins at the pressure p_s and ends at the pressure p_o. In this case the appropriate relations are taken into account but not considering the dryness fraction correction. The efficiency of all stages has been evaluated as follows:

$$\eta = \frac{\Delta h_{sh} + \Delta h_{sat}}{\Delta h_{is}} \tag{4.233}$$

$\Delta h_{sh}, \Delta h_{sat}$ being the enthalpy differences between the first and second lines expansion and Δh_{is} being the isentropic enthalpy difference between inlet and outlet pressures.

2. presence of regulation stage: referring to a body of steam turbine made of a partial admission stage, eventually followed by total admission stages.

If the isentropic enthalpy difference across the stage is lower (at least equal) than 52 kJ/kg, the peripheral speed is more or less 160 m/s (for velocity compounding stage) .The group consists only of the regulation stage. Otherwise, if the isentropic enthalpy difference is greater than 52 kJ/kg, the exceeding enthalpy difference is treated by the total admission stages. Two different cases are taken into account:

a) Presence of only regulation velocity compounding stage: the stage is designed with 5% exceeding mass flow than the nominal one. The inlet pressure drop is assumed of 5% due to the crossing of the steam through the regulation valves, assumed completely opened. Moreover $\frac{c_{is}^*}{u} = 2$ has been assumed, being:

$$c_{is}^* = \sqrt{2 \cdot \Delta h_{is}} \tag{4.234}$$

c_{is}^* being the steam velocity related to an isentropic expansion and u the inlet rotor row velocity. The basic value of the efficiency η_b is:

$$\eta_b = 91.05 - \frac{0.015533}{V_m^*} \tag{4.235}$$

$V_m^* \, [m^3 \cdot s^{-1}]$ being the volumetric flow in the designed condition $\dot{m} = \dot{m} \cdot (1+0.05)$.
Under reference conditions the inlet pressure loss is 6% (when the valves are partially closed), the isentropic enthalpy difference is evaluated. Therefore the velocity of the end of the isentropic expansion c_{is} and $\frac{c_{is}}{u}$ have been established:

$$\frac{c_{is}}{u} = \frac{\sqrt{2 \cdot \Delta h_{is}}}{c_{is}^* / 2} \tag{4.236}$$

The base efficiency has been corrected by introducing the coefficient c_1 that takes the row velocity u and the ratio $\frac{c_{is}}{u}$ into account:

$$c_1 = 4\left[\frac{u}{c_{is}} - \left(\frac{u}{c_{is}}\right)^2\right] \cdot \left[1 - 0.319 \cdot 10^{-2} \cdot \left(0.5 - \frac{u}{c_{is}}\right) \cdot (3.281 \cdot u - 250)\right] \tag{4.237}$$

The coefficient c_2 evaluated as a function of $\frac{c_{is}}{u}$,as figure 4.32 shows, contributes to reduce the efficiency.

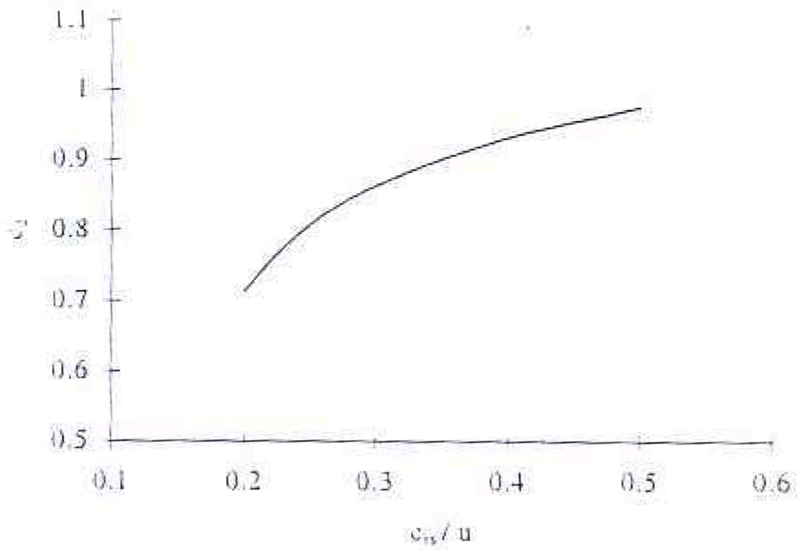


Fig. 4.32: regulation stage – losses corrective coefficient

If the isentropic line expansion is to be found partially or fully in the saturated region, a third coefficient c_3 is introduced. It takes the dryness fraction at the begin x_i and at the end x_{os} of the expansion:

$$c_3 = 1 - 0.87 \cdot \left(1 - \frac{x_i + x_{os}}{2}\right) \tag{4.238}$$

At least, considering all the corrective coefficients, the efficiency expression is presented below:

$$\eta_{reg} = \eta_b \cdot c_1 \cdot c_2 \cdot c_3 \tag{4.239}$$

2. Regulation stage followed by total admission stages: taking all the outlet quantities related to the regulated stage ($\eta_{reg}, h_{reg\_o}, p_{reg\_o}$) into account, the expansion till the outlet pressure p_o of the total admission stages is considered and correlation ad hoc for not regulated stages are taken into consideration to evaluate the efficiency and the outlet enthalpy h_u. The adiabatic efficiency is expressed as follows:

$$\eta = \frac{h_i - h_u}{\Delta h_{is}} \tag{4.240}$$

Δh_{is} being the isentropic enthalpy difference between inlet and outlet sections of the group.

4.3.11.3 model for the off-design analysis

Different models are taken to evaluate the steam turbine performances into consideration and the choice is related to the possibility or not of the presence of regulated partial admission stages.
If the group consists of regulated stages, the model will foresee the steam admission through infinite *valve points*. Maps expressing relation between pressure ratio β, steam mass flow $\dot{m}$, partialization ratio π and adiabatic efficiency η have been adopted:

$$F_1(\beta, \dot{m}, \pi) = 0 \tag{4.241}$$

$$F_1(\beta, \eta, \pi) = 0 \tag{4.242}$$

The maps are provided in *non dimensional form* and are scaled in relation to the referenced quantities $\beta^*, \dot{m}^*, \eta^*$.

The starting and the ending points of the expansion are linked with:

$$h_u = h_i - \eta \cdot (h_i - h_{os}) \tag{4.243}$$

h_{os} being the outlet isentropic enthalpy and s_i the inlet entropy evaluated by the constitutive equations:

$$f_s(p_i, h_i, s_i) = 0 \tag{4.244}$$

$$f_s(p_o, h_{os}, s_i) = 0 \tag{4.245}$$

and necessary calculation of the previous one.

On the contrary, if there are not regulated stages, the efficiency will be evaluated by assigning a generalized functional relation, also in this case are not dimensional one:

$$F_1(\beta, \eta) = 0 \tag{4.246}$$

The correlation between mass flows and pressures is established by Stodola's Low:

$$\frac{\mu}{\mu^*} = \sqrt{\frac{1-\left(\frac{p_o}{p_i}\right)^2}{1-\left(\frac{p_o^*}{p_i^*}\right)^2}} \qquad (4.247)$$

* being the referenced conditions, $\mu = m \cdot \frac{\sqrt{T}}{p}$ being the corrected mass flow.

The relation between the starting and ending point of expansion is established combining expressions (4.243), 4.244), (4.245).

Loss mechanical power is assumed to be proportional to the rotational speed *n*:

$$P_{mL} = P_{mL}^* \cdot \frac{n^*}{n} \qquad (4.248)$$

$n^*, P_{mL}{}^*$ being respectively the rotational speed and the loss mechanical power in the reference conditions, evaluated with $\eta_{mL} = 99\%$.

4.3.12 Pump

Figure 4.33 shows the generic pump scheme:

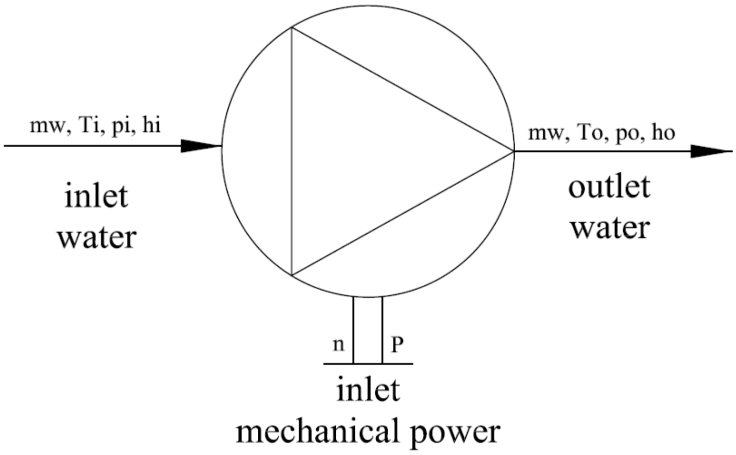


Fig. 4.33: scheme of a generic pump

Variables have been taken into considerations are:

<u>inlet water</u>

$\dot{m}_i\,[kg \cdot s^{-1}]$	water mass flow
$T_i\,[°C]$	temperature
$p_i\,[kPa]$	pressure
$h_i\,[kJ \cdot kg^{-1}]$	enthalpy

<u>inlet mechanical power</u>

$n\,[rpm]$	shaft rotational speed
$P_{mi}\,[MW]$	mechanical power

<u>outlet water</u>

$\dot{m}_w\,[kg \cdot s^{-1}]$	water mass flow
$T_o\,[°C]$	temperature
$p_o\,[kPa]$	pressure
$h_o\,[kJ \cdot kg^{-1}]$	enthalpy

The model is able to reproduce a fixed rotational speed pumps and variable speed ones.
Power consumption of the machine is expressed by the following relation:

$$P_{mi} = \frac{\dot{m}_w \cdot \Delta p}{\eta \cdot \rho_m} \tag{4.249}$$

Δp being the pressure increase between the inlet and the outlet section, ρ_m being the average density of the work fluid and η being the efficiency.
The enthalpy difference across the machine is evaluated as follows:

$$h_o - h_i = \frac{P_{mi}}{\dot{m}_w} \tag{4.250}$$

In the cycle calculation the efficiency value is set by the user.
For the off design operating conditions, two different cases have to be taken into account. One considering $n = cost$ and the other considering $n \neq cost$. In both cases the model takes the characteristic curves that give the manometric prevalence and the efficiency versus the mass flow and velocity into account.
Referring to a constant rotational speed of the shaft, these curves are expressed as the product of the nominal value for the normalized value related to it. Expressing the normalized mass flow $\mu = \frac{\dot{m}}{\dot{m}^0}$, the prevalence Π and the efficiency η, they are expressed by the following relations:

$$\Pi = \Pi^0 \cdot f_1(\mu) \tag{4.251}$$

$$\eta = \eta^0 \cdot f_2(\mu) \tag{4.252}$$

$(^0)$ being representative of the nominal conditions, $f_1(\mu), f_2(\mu)$ being representative of the normalized curves previously described.
In the case of variable rotational speed, the curves depend on the normalized velocity defined as follows:

$$\nu = \frac{n}{n^0} \tag{4.253}$$

and are expressed as a function of the both variables $f_1(\mu,\nu), f_2(\mu,\nu)$. These functions are tabulated for different normalized speed and during the calculations the most adapted curve is selected. Thus, operating in kinematic similarity, efficiency and prevalence curves are determined vs the actual mass flow and actual velocity.

4.3.13 Deaerator

Figure 4.34 shows the deaerator model scheme:

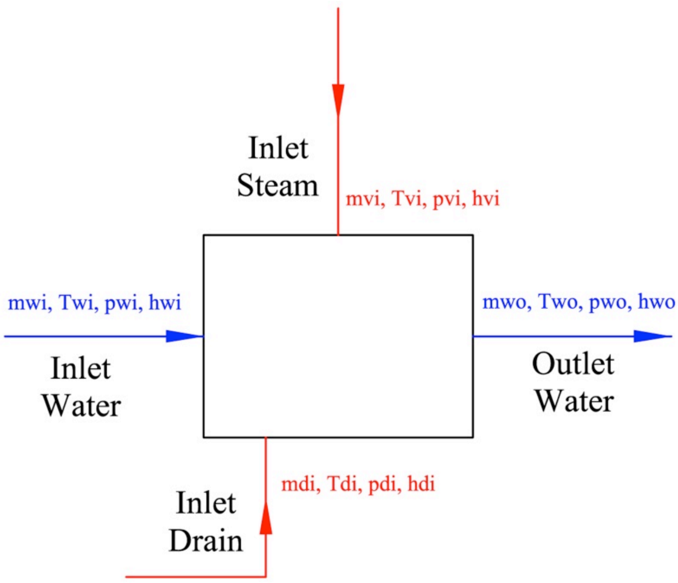


Fig. 4.34: scheme of deaerator

Variables have been taken into considerations are:

inlet water

$\dot{m}_{wi}\,[kg \cdot s^{-1}]$ water mass flow

$T_{wi}\,[°C]$ temperature

$p_{wi}\,[kPa]$ pressure

$h_{wi}\,[kJ \cdot kg^{-1}]$ enthalpy

inlet steam

$\dot{m}_{vi}\,[kg \cdot s^{-1}]$ water mass flow

$T_{vi}\,[°C]$ temperature

$p_{vi}\,[kPa]$ pressure

$h_{vi}\,[kJ \cdot kg^{-1}]$ enthalpy

inlet water

$\dot{m}_{di}\,[kg \cdot s^{-1}]$ water mass flow

$T_{di}\,[°C]$ temperature

$p_{di}\,[kPa]$ pressure

$h_{di}\,[kJ \cdot kg^{-1}]$ enthalpy

outlet water

$\dot{m}_{wo}\,[kg \cdot s^{-1}]$ water mass flow

$T_{wo}\,[°C]$ temperature

$p_{wo}\,[kPa]$ pressure

$h_{wo}\,[kJ \cdot kg^{-1}]$ enthalpy

The model takes the mixing of the different fluxes at the pressure p_{vi} into account. The model is based on conservation low of mass,

$$\dot{m}_{wo} = \dot{m}_{wi} + \dot{m}_{vi} + \dot{m}_{di} \tag{4.254}$$

on conservation low of energy:

$$\dot{m}_{wo} \cdot h_{wo} = \dot{m}_{wi} \cdot h_{wi} + \dot{m}_{vi} \cdot h_{vi} + \dot{m}_{di} \cdot h_{di} \tag{4.255}$$

and on the constitutive equations of the fluid.
A constraint, considering the equality of the water outlet enthalpy h_{wo} to the saturation enthalpy $h_{sat}(p_i)$, has been taken into account:

$$h_{wo} = h_{sat}(p_i) \tag{4.256}$$

4.3.14 Attemperator

Figure 4.35 shows the attemperator (desuperheating steam) model scheme:

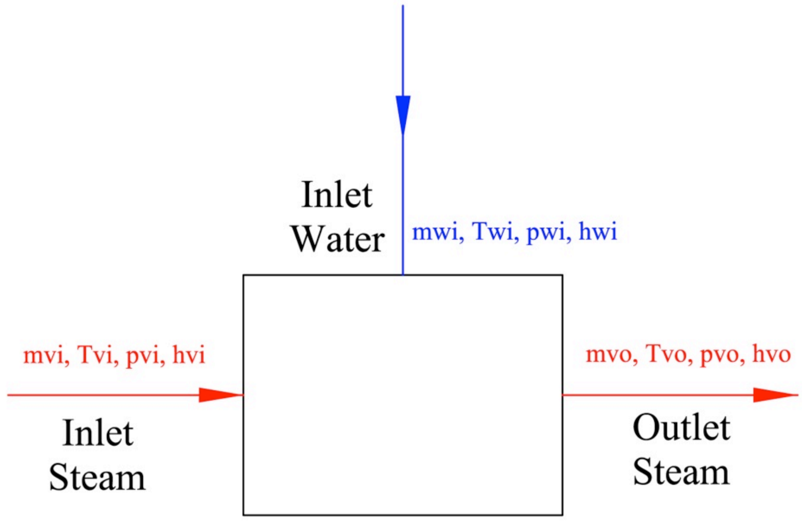


Fig. 4.35: scheme of the attemperator

Variables have been taken into considerations are:

<u>inlet water</u>

$\dot{m}_{wi}\ [kg \cdot s^{-1}]$	water mass flow
$T_{wi}\ [°C]$	temperature
$p_{wi}\ [kPa]$	pressure
$h_{wi}\ [kJ \cdot kg^{-1}]$	enthalpy

<u>inlet steam</u>

$\dot{m}_{vi}\ [kg \cdot s^{-1}]$	water mass flow
$T_{vi}\ [°C]$	temperature
$p_{vi}\ [kPa]$	pressure
$h_{vi}\ [kJ \cdot kg^{-1}]$	enthalpy

<u>outlet steam</u>

$\dot{m}_{vo}\ [kg \cdot s^{-1}]$	water mass flow
$T_{vo}\ [°C]$	temperature
$p_{vo}\ [kPa]$	pressure
$h_{vo}\ [kJ \cdot kg^{-1}]$	enthalpy

The model takes the mixing of the different fluxes at the pressure p_{vi} into account. The model is based on conservation low of mass,

$$\dot{m}_{vo} = \dot{m}_{wi} + \dot{m}_{vi} \qquad (4.257)$$

on conservation low of energy:

$$\dot{m}_{vo} \cdot h_{vo} = \dot{m}_{wi} \cdot h_{wi} + \dot{m}_{vi} \cdot h_{vi} \tag{4.258}$$

and on the constitutive equations of the fluid.

4.3.15 Junctions

The following model describes the mixing of two material flows. The developed model takes mixing of gas flows, water (steam) and water with gas into account. The scheme of the generic model is shown in the following figure 4.36:

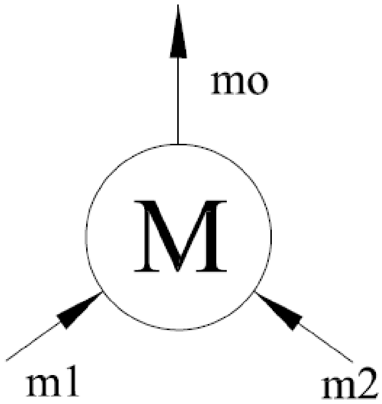


Fig. 4.36: scheme of the mixer

The model is based on conservation low of mass,

$$\dot{m}_{o} = \dot{m}_{1} + \dot{m}_{2} \tag{4.259}$$

on conservation low of energy:

$$\dot{m}_{o} \cdot h_{o} = \dot{m}_{1} \cdot h_{1} + \dot{m}_{2} \cdot h_{2} \tag{4.260}$$

and on the constitutive equations of the fluid.

m being mass flow rate, h enthalpy and where subscripts *1*, *2*, *o* refer to first, second incoming flows and to the exiting flow, respectively.

4.3.16 Derivation

The developed model refers to ramification gas flows, water (steam) and fuel, figure 4.37.

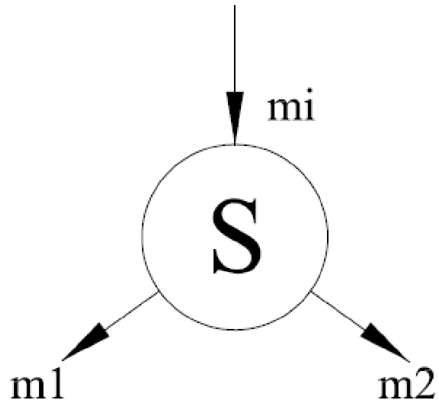


Fig. 4.37: scheme of the derivation

The model is based on conservation low of mass,

$$\dot{m}_i = \dot{m}_1 + \dot{m}_2 \tag{4.261}$$

m_i being incoming flow rate, m_1 and m_2 exiting flow rates.
Fraction of divided flows can be assigned; in this case, further condition is applied:

$$\dot{m}_1 = \varphi \cdot \dot{m}_i \tag{4.262}$$

4.3.17 Pressure loss devices

The model is used to describe the devices that establish a pressure difference between two domains. Models for devices operating with gas and water flows are taken into account.
The fluid evolves according to an isenthalpic transformation. Inlet pressure p_i and outlet pressure p_u are correlated as follows:

$$p_o = p_i \cdot (1 - k_p) \tag{4.263}$$

k_p being loss coefficient.

In the cycle calculation, the previous coefficient can be either assigned or not. Two different cases can be distinguished: device with fixed or variable opening.
In the first case, the flow rate is correlated to the pressures at the extremities of the device as follows:

$$\mu^2 = k \cdot \frac{p_i - p_o}{p_i} \tag{4.264}$$

being

$$\mu = \frac{m \cdot \sqrt{T_i}}{p_i} \tag{4.265}$$

the corrected inlet flow rate and k being a constant obtained with respect to reference values: μ^0, p_i^0 e p_u^0.
In the second case, device opening is automatically adapted in order to maintain the controlled variables (pressure or flow rate) at the assigned values.

4.3.18 Electric generator

The model scheme is reported in the following figure:

At inlet and outlet power, the following variables are assigned:

- mechanical power inlet

$P_{mi}\,[MW]$	mechanical power
$n\,[rpm]$	shaft rotational speed

- electric power outlet

$P_e\,[MW]$	electric power
$n\,[rpm]$	shaft rotational speed

The electric power can be written as:

$$P_e = P_m - P_{ml} - P_{el} \tag{4.266}$$

P_{ml}, P_{el} mechanical and electric power losses respectively.

In the cycle calculation, the previous quantities are estimated giving the electric and mechanical efficiencies. During the elaboration phase, the electric P_{el}^* and the mechanical P_{ml}^* losses are calculated in reference conditions.

The following data are requested:

- $n_g\,[rpm]$ generator rotational speed
- $n_m\,[rpm]$ motor rotational speed
- $P_e^*\,[MW]$ reference electric power

The lost electric power is calculated in function of the electric power P_e^*:

$$P_{el}^* = 0.01 \cdot P_e^* \cdot (A - 0.34293488 + 0.89126466 \cdot \phi_e - 0.1087785 \cdot \phi_e^2 + 0.0036908606 \cdot \phi_e^3) \tag{4.267}$$

$\phi_e = \ln P_e^*$, P_{el}^* and P_e^* are expressed in kW, A is Zero if the generator velocity is equal to 3000 rpm. For values inferior to 3000 rpm, A= -0.095.

The mechanical power loss $P_{m\lg}^*$ (kW) is expressed in function of the reference electric power P_e^* (kW):

$$P_{m\lg}^* = 0.29898103 + 0.00294889 \cdot P_e^* + 0.612245 \cdot 10^{-9} \cdot P_e^{*2} \tag{4.268}$$

If n_m is different from n_g, the velocity is reduced. The associated mechanical power loss is calculated as:

$$P_{mlr}^* = -(P_e^* + P_{m\lg}^*) \cdot \left(1 - \frac{1}{\eta_r}\right) \tag{4.269}$$

$$\eta_r = 8.889 \cdot 10^{-8} \cdot P_e^* + 0.989 \qquad \text{for} \qquad P_e^* \leq 100.10 \cdot 10^3 \; kW$$

$$\eta_r = 0.998 \qquad \text{for} \qquad P_e^* > 100.10 \cdot 10^3 \; kW$$

The global mechanical power loss of the electric generator is expressed as follows:

$$P_{ml}^* = P_{m\lg}^* + P_{mlr}^* \tag{4.270}$$

In part load condition, the lost mechanical power is assumed as constant and equal to the reference value P_{ml}^*.

The lost electric power P_{el} is related to the produced electric power P_e with the empirical relationship:

$$P_{el} = k \cdot \left(\frac{P_{el}^*}{P_e^*} \right) \cdot P_e \tag{4.271}$$

P_{el} and P_e are expressed in kW, and K coefficient is given as:

$$k = \frac{-20.525271 - 35.019 \cdot \left(\frac{P_e}{P_e^*} \right) + 25.514 \cdot \left(\frac{P_e}{P_e^*} \right)^2 - 25.1824 \cdot \left(\frac{P_e}{P_e^*} \right)^3}{1 - 56.218495 \cdot \left(\frac{P_e}{P_e^*} \right)} \tag{4.272}$$

Since, for very low values of the electric power P_e, the k value is unreliable. So, the following control is introduced: for the limit value of P_e, k=1.

5. Models adapted for the H2-IGCC Gas Turbine

5.1 Introduction

H2-IGCC project is based on a Power Island that contains a Gas Turbine and a bottomer steam cycle connected also to the Gas Production section. The gas turbine class has been assumed to be a *F* one the power being about 300MW. It is expected to have a generic F class 300MW GT like the 94.3A Ansaldo and SGT5 – 4000F Siemens engines.
To produce the GT model the following main modules have been adapted:

- Compressor
- Expander
- Combustion Chamber

The physical already defined modules have been adapted to have the modules to be matched together using also the minor component models such as valve, splitter, junction, shaft, etc. to obtain the **G**eneric **F** **C**lass **300**MW **GT** (*GFC300GT)* that is schematically given in figure 5.1:

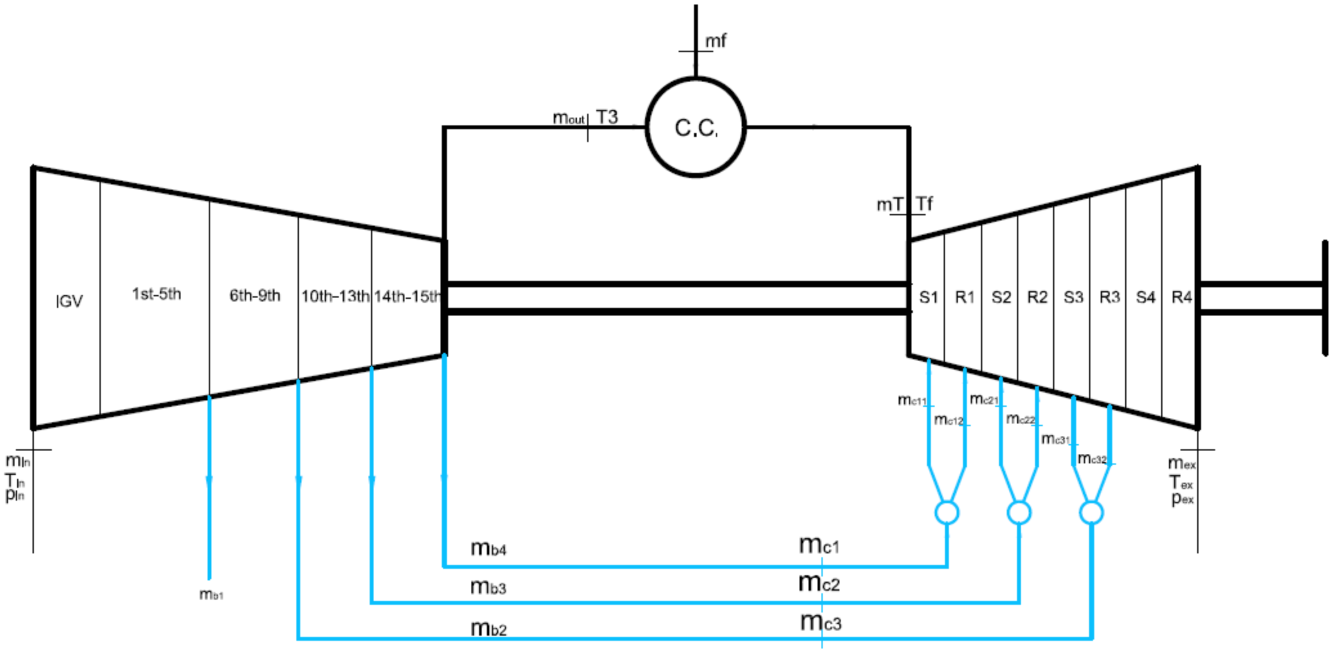


Fig. 5.1: Generic F class 300MW GT

The GT through flow section is reported in fig. 5.2. Non dimensional data representing the H2IGCC 300 MW F Class GT required to adapt the models to the H2–IGCC GT are given in the APPENDIX E tables. Data are referred to the size of the VIGV Hub Diameter.

- reference radius $r_0 = 692.5\,mm$

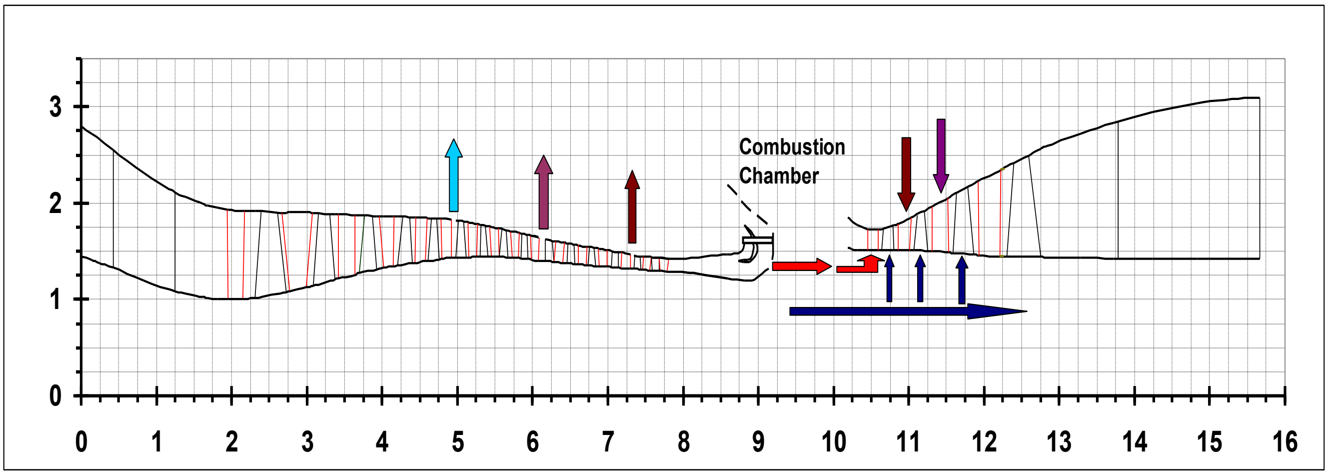


Fig. 5.2: compressor and expander non-dimensional through flow shape

5.2 Compressor

The block scheme of the H2-IGCC 300 MW F Class GT compressor adapted model is given to figure 5.3 where four compressor bodies have been taken into consideration to take into account of the bleeds.

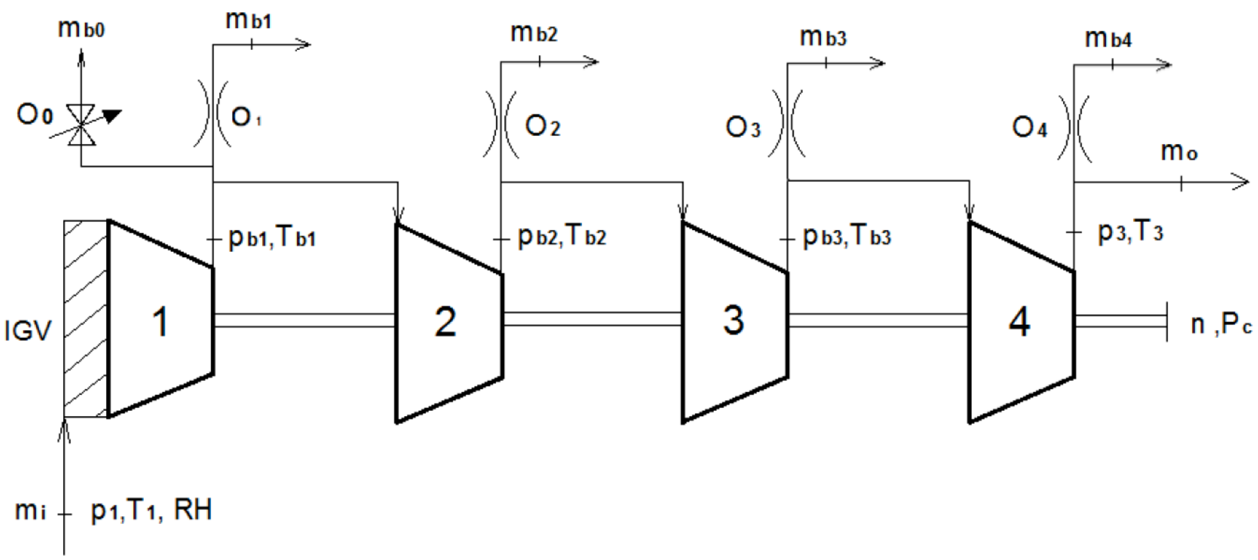


Fig. 5.3: compressor adapted model scheme

Nominal condition data are given in table 5.1

Table 5.1: compressor data

Inlet pressure	p_1	101,3	kPa
Inlet temperature	T_1	15,0	°C
Relative humidity	RH	60	%
1st bleed mass flow	m_{b1}	2	[%mi]
2nd bleed mass flow	m_{b2}	3	[%mi]
3rd bleed mass flow	m_{b3}	7	[%mi]
4th bleed mass flow	m_{b4}	3	[%mi]
Exit mass flow rate	m_3	582	kg/s
Exit pressure	p_3	1844	kPa
Exit temperature	T_3	397	°C
Power	P_c	264	MW

The compressor blade to blade scheme with the lumped blade profiles are given in figure 5.4 and data in the APPENDIX F.

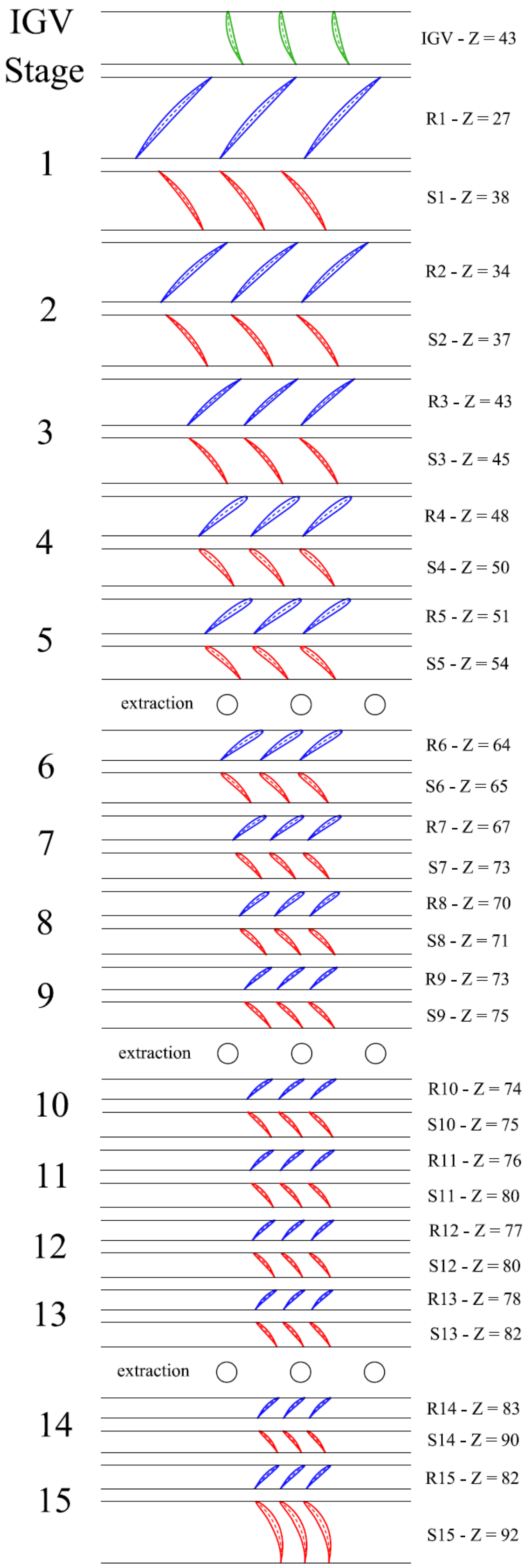


Fig. 5.4: Compressor blade to blade overview

5.3 Combustion Chamber

Tables 5.2 and 5.3 show data related to the combustion chamber adapted model at nominal conditions

Table 5.2: combustion chamber data

INPUT QUANTITIES				
1	Inlet Mass flow Rate	mgi	514	kg/s
2	Inlet Pressure	pgi	1844	kPa
3	Inlet Temperature	Tgi	397	°C
4	Inlet Fuel Mass Flow Rate	mf	14.5	kg/s
5	Low Heating Value (LHV)	PCI	49736	kJ/kg
6	Expected Flame Temperature	Tflame	1440	°C
7	Pressure Drop [%]	Δpcc	5.6	%
8	Air Composition	[N2]	76.3	$\%_m$
		[O2]	23.1	$\%_m$
		[H20]	0.0	$\%_m$
		[CO2]	0.6	$\%_m$
9	Fuel Composition	[CH4]	100	$\%_m$
		[H2]	0.0	$\%_m$
		[CO]	0.0	$\%_m$
		[CO2]	0.0	$\%_m$
		[O2]	0.0	$\%_m$
		[N2]	0.0	$\%_m$
		[H2O]	0.0	$\%_m$

Table 5.3: combustion chamber data

OUTPUT QUANTITIES				
1	Exit Mass flow rate	mgo	528.3	kg/s
2	Exit Pressure	pgo	1788	kPa
3	Exit Temperature	Tgi	1440	°C
4	Exhaust Gas Composition	[N2]	74.2	$\%_m$
		[O2]	11.5	$\%_m$
		[H20]	6.8	$\%_m$
		[CO2]	7.5	$\%_m$
5	Combustion Chamber Efficiency (ref)	η	95.0	%

5.4 Gas Expander

The gas expander has been modelled according to figure 5.5:

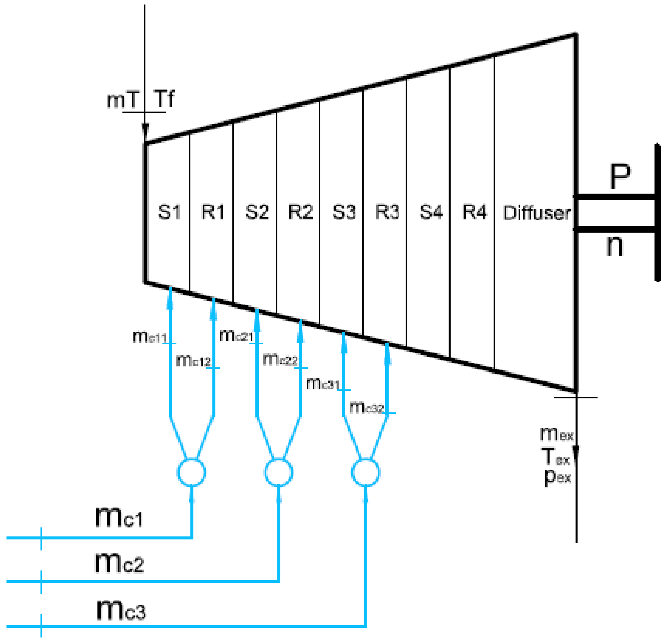


Fig. 5.5: gas expander adapted model scheme

Cooling flows normalized in respect to the compressor inlet mass flow, cooling effectiveness and blade temperatures normalized in respect to the firing temperature at the expander design point are given in table 5.4. Data at nominal conditions are given in table 5.5.

Table 5.4: Blade cooling data at the GT nominal point

	mc [%]	eff [#]	Tb/Tf [#]
1S	6.3	0.58	0.62
1R	5.8	0.53	0.61
2S	4.4	0.53	0.57
2R	3.4	0.46	0.55
3S	1.8	0.40	0.54
3R	2.3	0.29	0.53
4S	-	-	-
4R	-	-	-
DIFFUSER	-	-	-

Table 5.5: Gas expander design point

Exhaust Mass flow	m2	686.8	kg/s
Exhaust Temperature (static)	T2	568.0	°C
Exhaust Temperature (total)	T20	577.6	°C
Exhaust Pressure (static)	p2	104.2	kPa
Exhaust Pressure (total)	p20	108.6	kPa
Power	Pe	569.3	MW

The gas expander lumped blade to blade section is given in fig. 5.6:

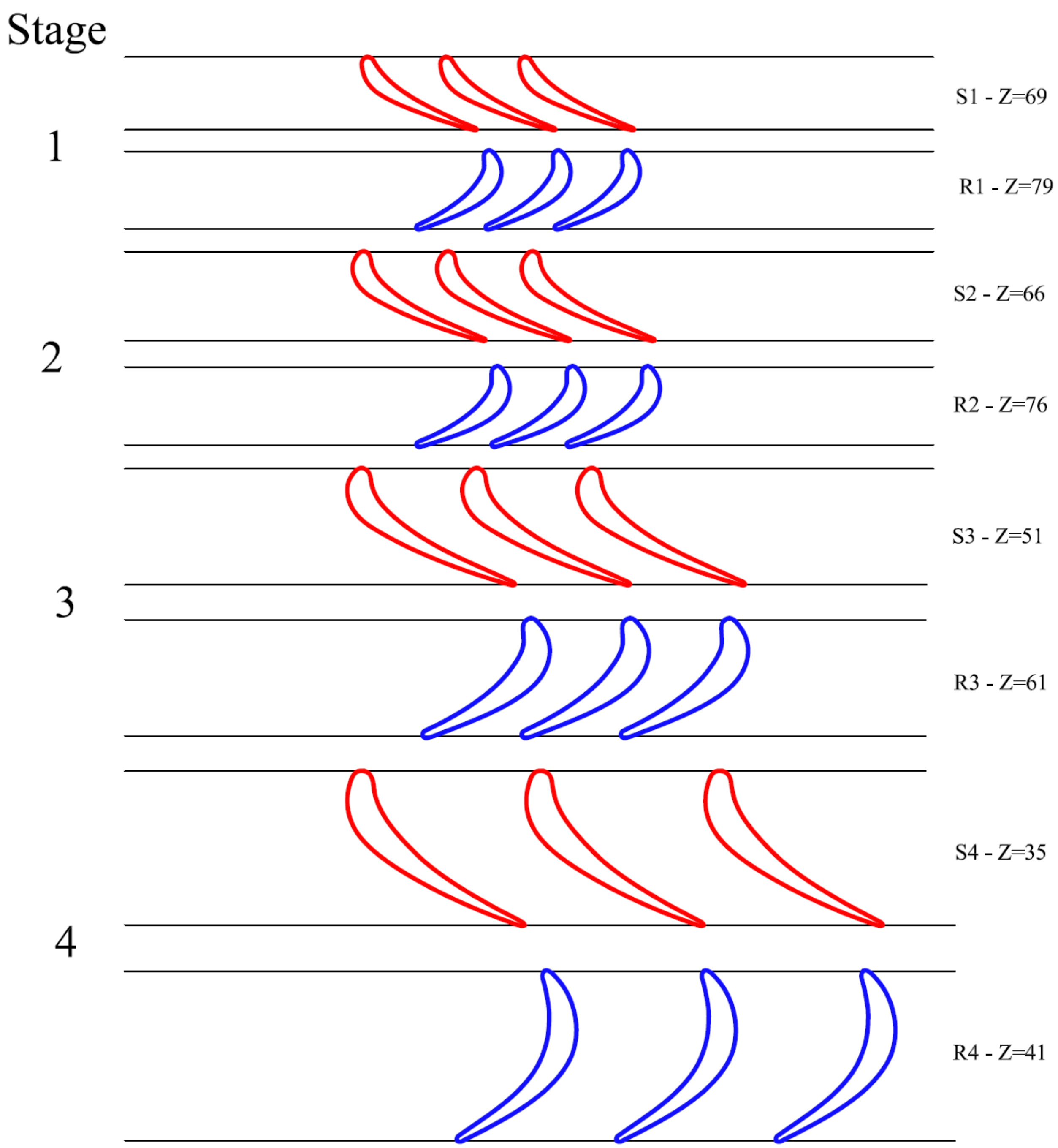


Fig. 5.6: Gas expander lumped blade to blade section

Data used to obtain the lumped blade to blade section are given in APPENDIX G.

6. Matching

Matching all the components described in previous paragraphs, a generic GT simulator has been achieved. Figure 6.1 shows the through flow shape of the whole GT.

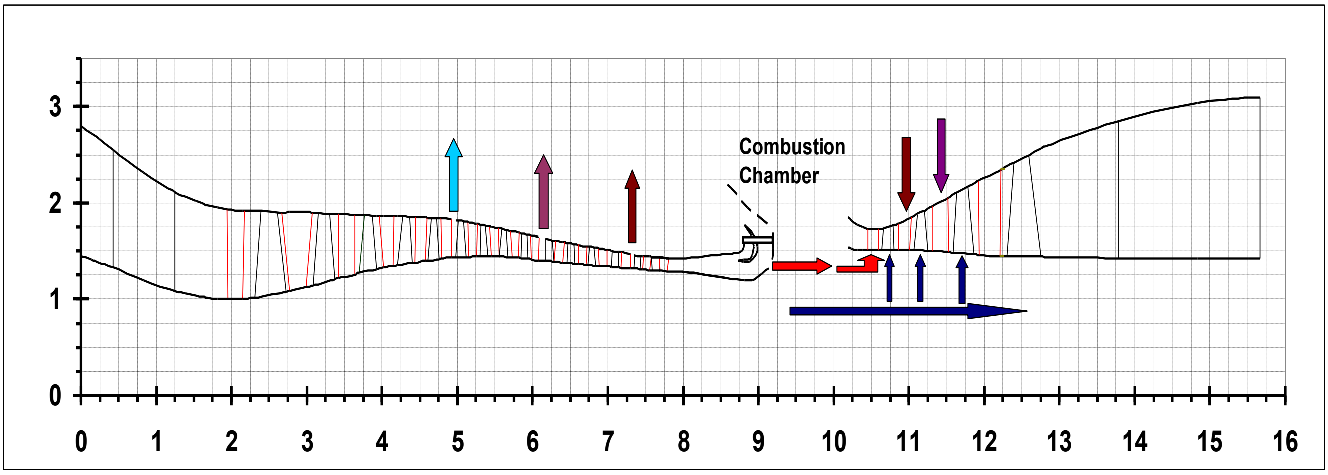


Fig. 6.1: Gas Turbine Through Flow Shape

All component modules are connected together among the flow, electric and mechanical connections. All the modules have been sized previously according to their design and boundary conditions. The equilibrium behaviour is searched by a simultaneous solution approach taking the control quantities set at their nominal values. Calculation objective is aimed at establishing all the connection related quantities such as openings, exhaust mass flow, temperature, etc. Degree of freedom have been VIGV percentage and nominal firing temperature. A flow diagram showing the connections among the components is given in Figure 6.2.

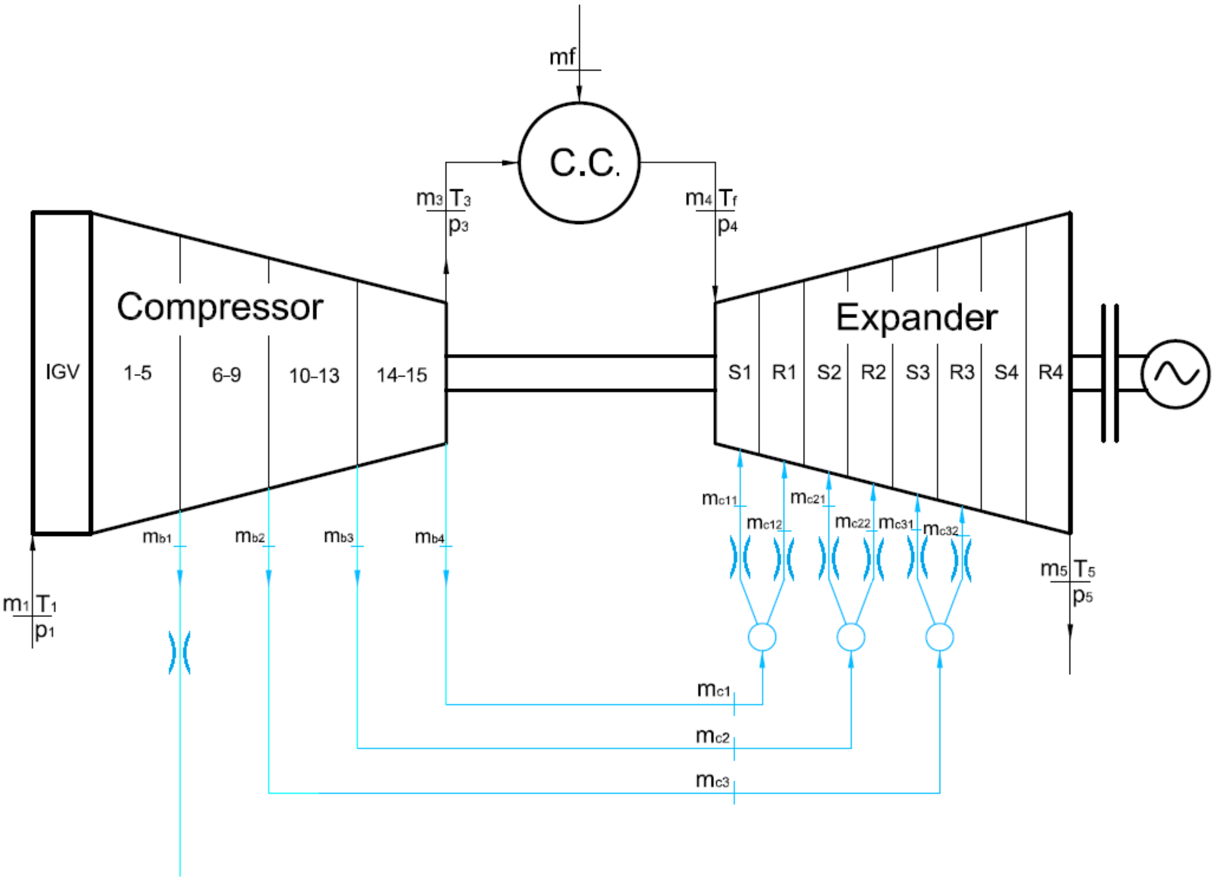


Fig. 6.2: GT scheme

Component models used in this Matching calculations are those for the sized component off-design analyses. A schematic view of the calculation approach is shown in Figure 6.3.

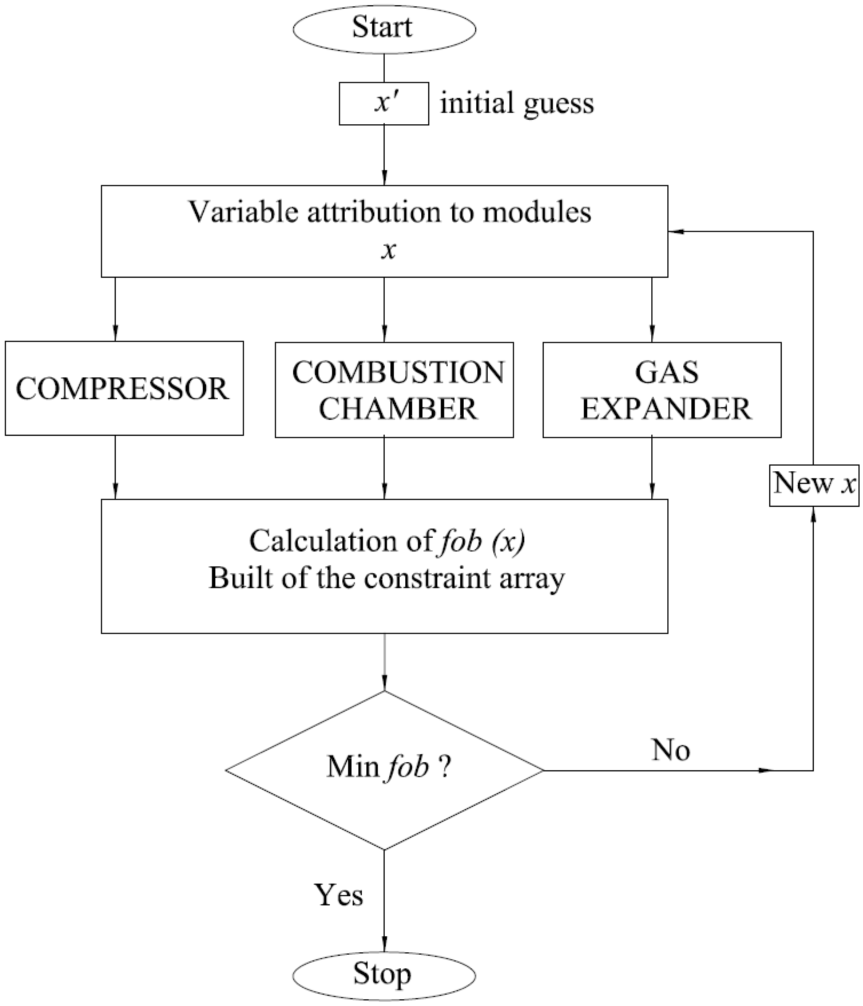


Figure 6.3: Calculation approach

6.1 Input quantities and boundary conditions

With reference to the first paragraph of this document, the set of equation *F* contains variables, degree of freedoms, geometrical data and boundary conditions. Taking the matching of compressor, combustion chamber and gas expander into consideration, the quantities below have been assumed to the simulator in order to obtain the unknown quantities of interest (i.e. pressures, temperatures, etc.).

Table 6.1: Nominal input quantities of the GT matching problem

AMBIENT CONDITIONS (ISO)	
[kPa]	p inlet compressor
[°C]	t inlet compressor
[%]m	Air Composition
[%]	RH
COMPRESSOR	
[%]	VIGV Opening
COMBUSTION CHAMBER	
[%]m	Fuel Composition
[%]	Pressure Drop
[%]	Efficiency
GAS EXPANDER	
[°C]	t firing
[°C]	t coolant
[kPa]	p outlet diffuser (static)
GAS TURBINE	
[MW]	Mechanic Power Loss
[MW]	Electric generator Power Loss
[rpm]	Rotational Speed

6.2 Output quantities

The output given by the GT simulator have been determined by the matching of the components. That means outlet quantities of the compressor (pressure, temperature, etc.), are the input quantities of the combustion chamber. They are the same variables and are called *matching variables*.
Table 6.2 shows the performances of the GT at the nominal operating point with the NG feeding.

Table 6.2: Gas Turbine matching main quantities

Gas Turbine Performances	
COMPRESSOR	
[%]	VIGV OPENING
[deg]	VIGV ANGLE
[#]	PRESSURE RATIO
[%]	EFFICIENCY
[MW]	POWER
COMBUSTION CHAMBER	
[kg/s]	INLET FUEL
[MJ/kg]	LHV
[%]	CC EFFICIENCY
[%]	PRESSURE LOSS
[MW]	POWER INTRODUCED BY FUEL
GAS EXPANDER	
[°C]	TiT ISO 2314
[%]	TOTAL TO TOTAL EFFICIENCY
[%]	TOTAL TO STATIC EFFICIENCY
[MW]	MECHANICAL POWER
GAS TURBINE	
[MW]	MECHANICAL POWER
[%]	EFFICIENCY
[kCal/kWh]	SPECIFIC CONSUMPTION

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